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Identifying Monetary Policy Shocks via Heteroskedasticity: a Bayesian Approach

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Identifying Monetary Policy Shocks via Heteroskedasticity: a Bayesian Approach

Dmitry Kulikov and Aleksei Netšunajev*

Abstract

In this paper we contribute to the literature on the identification of macroeconomic shocks by proposing a Bayesian SVAR with time-varying volatility of innovations that depend on a hidden Markov process, referred to as an MS-SVAR. With sufficient statistical information in the data, the distinct volatility regimes of the errors allow all the structural SVAR matrices and impulse response functions to be identified without the need for conventional *a priori* parameter restrictions. We give mathematical identification conditions and propose a flexible Gibbs sampling approach for the posterior inference on MS-SVAR parameters. The new methodology is applied to the US, euro area and Estonian macroeconomic series, where the effects of monetary policy and other shocks are examined.

JEL Code: C11, C32, C54

Keywords: Markov switching model, volatility regimes, Bayesian inference, monetary policy shocks, SVAR analysis

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Non-technical summary

Structural vector autoregressions are popular tools of modern empirical macroeconomics, widely used in monetary policy analysis and other applications to examine dynamic interactions and the effects of various shocks on real and nominal macroeconomic aggregates. A central issue of this research is the identification of structural shocks, such as monetary policy, aggregate demand and aggregate supply shocks, where the conventional approach used in most empirical research in the area requires restrictions to be placed directly on the structural vector autoregression parameters. However, recent literature has raised some criticism of the conventional approach: even if the identifying assumptions imposed are based on a widely accepted economic idea, there may still be a gap between the data and the theoretical model, leading to a potentially biased inference from the dynamic reactions of model variables and confounding the development of new theories.

In this study we depart from the traditional shock identification approach and use additional statistical information, available in many macroeconomic data series as the time-varying volatility of error terms, to help identify structural parameters and interpret shocks. We model the volatility states of the errors using a hidden Markov process, referring to the new framework as the Markov switching structural vector autoregression. We show how existing mathematical results allow a statistical identification of the structural parameters in at least two volatility regimes, without the need for any a priori identifying assumptions to be imposed. We apply Bayesian statistical inference for parameter estimation and shock identification in the new framework.

The new methodology is validated using the medium-scale monetary policy systems of the USA and the euro area, and a small-scale model with an interest rate premium for the Estonian economy. Previous empirical research has shown that the US macroeconomic data since the mid-1960s have been characterised by the time-varying volatility of macroeconomic shocks, while the remaining model parameters can be considered stable. A similar set of results is gained from the euro area macroeconomic data starting from the early 1970s. We find sufficient volatility information in our data samples to be able to identify and disentangle a full set of shocks for every estimated model in our empirical applications. Furthermore, we undertake a careful economic interpretation of the identified shocks by looking at their short-run impacts and impulse responses, comparing them with the existing literature and finding consistent economic narratives for every shock in our empirical models. The shock identification in our models is achieved without the a priori identifying restrictions that are common in other empirical studies. Although we are mostly interested in the monetary policy and risk premium shocks, our

statistical identification methodology enables us simultaneously to disentangle and attach economic interpretations to other structural macroeconomic innovations, such as aggregate demand, aggregate supply, and money demand shocks.

We also point out that the results of our statistical shock identification procedure are not always compatible with the traditional short run and sign identification schemes used in much of the recent empirical literature, which warrants further careful validation and checking of the existing results using the new identification methodology in this paper as well as other alternative approaches.

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1. Introduction

Structural vector autoregressions (SVARs) are well-known tools of modern empirical macroeconomics, widely used in monetary policy analysis and other applications to examine dynamic interactions and the effects of various shocks on real and nominal macroeconomic aggregates. A central issue in SVAR analysis is the reliable and robust identification of structural shocks, where the conventional approach used in most of empirical research in the area calls for restrictions to be placed on the SVAR parameters that deliver the statistical identification of the model; see Sims (1980), Blanchard and Quah (1989), Amisano and Giannini (1997), Canova and De Nicolo (2002), among many others. However, recent literature has raised some criticism of the just-identifying and sign restrictions, detailed in Lütkepohl (2012) and Lütkepohl and Netšunajev (2013). Even if the imposed identifying assumptions are based on a widely accepted economic idea, there may still be a gap between the data and the theoretical model, leading to a potentially biased inference on the dynamic reactions of model variables and confounding the development of new theories.

In this study we use additional statistical information, available in many macroeconomic data series in the form of time-varying volatility of VAR error terms, to help with the identification of structural parameters and the interpretation of shocks within a modified SVAR framework along the lines of Rigobon (2003) and Lanne and Lütkepohl (2008). We model the volatility states of the errors by a hidden Markov process, similar to that used in Lanne, Lütkepohl and Maciejowska (2010), referring to the overall framework as an MS-SVAR. We show how existing mathematical results enable a statistical identification of SVAR structural matrices in the case of at least two volatility regimes, without the need to impose any *a priori* identifying assumptions.

Recently, a handful of applied studies have been published on shock identification in SVAR models through volatility regimes using the maximum likelihood estimator; see Bacchiocchi and Fanelli (2012) and Netšunajev (2013), among others. In the present study we depart from the maximum likelihood approach and resort to the Bayesian statistical inference for parameter estimation and shock identification in the MS-SVAR framework. The latter has several advantages over the classical approach: it enables us to get a full picture of the posterior effects of different shocks, proving invaluable in validating the identifying assumptions and finding convincing economic narratives for the observed structural shocks in estimated models. Furthermore, the Bayesian approach is less sensitive to likelihood function irregularities and numerical maximization complexities, which can be a particularly daunting issue in the hidden Markov models; see Herwartz and Lütkepohl (2011). At the same time, Bayesian inference requires the postulation of suitable priors, depends on intricate numerical algorithms and tends to be computationally demanding.

Our empirical applications are focused on the medium-scale monetary policy VARs for the US and the euro area (EA), and a small-scale VAR model with an interest rate premium for the Estonian economy. As pointed out by Primiceri (2005) and Sims and Zha (2006), the US macroeconomic data since the mid-1960s have been characterized by the time-varying volatility of macroeconomic shocks, while the remaining reduced-form system parameters can be considered stable. A similar set of results is reported in Rubio-Ramirez, Waggoner and Zha (2005) for the EA macroeconomic data series starting from the early 1970s. We find sufficient volatility information in our data samples to be able to identify and disentangle a full set of shocks for every estimated model in our empirical application. Furthermore, we undertake a careful economic interpretation of the identified shocks by looking at their posterior short-run impacts and impulse responses, comparing them with the existing literature and finding consistent economic narratives for every shock in our empirical models. The shock identification in our empirical MS-SVAR models is achieved without the *a priori* identifying restrictions that are common in the empirical SVAR studies. Although we are mostly interested in the monetary policy and risk premium shocks, our statistical identification methodology enables us simultaneously to disentangle and attach economic interpretations to other structural macroeconomic innovations, such as aggregate demand, aggregate supply, and money demand shocks.

The rest of the paper is organized as follows. Section 2 describes the MS-SVAR model and the identification of its parameters through the time-varying volatility of the innovations. Section 3 gives a detailed overview of the Gibbs sampler-based Bayesian inference for the new model. Section 4 presents three empirical applications of the MS-SVAR model and illustrates the practical shock identification issues: the model is applied to samples of the US, the EA and Estonian data, which cover different time periods and have vastly different amounts of statistical information in the estimated volatility regimes. Finally, the last section concludes and proposes some potential directions for future research.

2. Econometric specification and model identification

Let the time evolution of an $n \times 1$ vector $\boldsymbol{y}_t$ of endogenous variables be given by the following SVAR model:

$$\mathbf{A}_0 \boldsymbol{y}_t = \boldsymbol{k}_0 + \boldsymbol{k}_1 t + \mathbf{A}_1 \boldsymbol{y}_{t-1} + \ldots + \mathbf{A}_p \boldsymbol{y}_{t-p} + \boldsymbol{\epsilon}_t(s_t)\,, \tag{1}$$

where $\boldsymbol{k}_0$ and $\boldsymbol{k}_1$ are (optional) deterministic intercept and linear trend parameters respectively, $\mathbf{A}_0$ is a general $n \times n$ contemporaneous parameter matrix, $\mathbf{A}_1, \ldots, \mathbf{A}_p$ are autoregressive matrices, and $\boldsymbol{\epsilon}_t(s_t)$ is a vector of serially uncorrelated structural innovations that depends on the hidden state parameter $s_t \in \{1, \ldots, m\}$. We assume the following conditional distribution of the structural innovations:

$$\boldsymbol{\epsilon}_t(s_t) \,|\, s_t \sim \text{Normal}(\,\mathbf{0}, \mathbf{D}(s_t)\,)\,,$$

where $\{\mathbf{D}(s) : 1 \leq s \leq m\}$ is a family of distinct $n \times n$ diagonal matrices, where $\mathbf{D}(1) \equiv \mathbf{I}$ is imposed for identification. When $m = 1$, this model reduces to the textbook SVAR case; see Hamilton (1994), Amisano and Giannini (1997) and Lütkepohl (2005). When the hidden state parameter s_t is Markov and the number of states is greater than one, we call the model (1) a "Markov-switching structural vector autoregression" (MS-SVAR).

Assuming that the contemporaneous parameter matrix $\mathbf{A}_0$ is non-singular, the model can be written in the usual reduced-form VAR style with time-varying volatility of errors:

$$\boldsymbol{y}_t = \boldsymbol{c}_0 + \boldsymbol{c}_1 t + \boldsymbol{\Phi}_1 \boldsymbol{y}_{t-1} + \ldots + \boldsymbol{\Phi}_p \boldsymbol{y}_{t-p} + \boldsymbol{u}_t(s_t)\,, \tag{2}$$

where $\boldsymbol{c}_i = \mathbf{A}_0^{-1} \boldsymbol{k}_i$ for each $0 \leq i \leq 1$, $\boldsymbol{\Phi}_j = \mathbf{A}_0^{-1} \mathbf{A}_j$ for each $1 \leq j \leq p$, and:

$$\boldsymbol{u}_t(s_t) \,|\, s_t \sim \text{Normal}(\,\mathbf{0}, \boldsymbol{\Sigma}(s_t)\,)\,,$$

where $\boldsymbol{\Sigma}(s) = \mathbf{A}_0^{-1} \mathbf{D}(s) \mathbf{A'}_0^{-1}$ for each volatility state $s \in \{1, \ldots, m\}$, giving rise to the family of reduced-form variance-covariance matrices $\{\boldsymbol{\Sigma}(s) : 1 \leq s \leq m\}$. Model (2) forms the basis for writing down the likelihood function of the sample data in Section 3.

Consider model (1) premultiplied by an arbitrary conformable unitary matrix $\mathbf{U}$:[1]

$$\mathbf{U}\mathbf{A}_0 \boldsymbol{y}_t = \mathbf{U}\boldsymbol{k}_0 + \mathbf{U}\boldsymbol{k}_1 t + \mathbf{U}\mathbf{A}_1 \boldsymbol{y}_{t-1} + \ldots + \mathbf{U}\mathbf{A}_p \boldsymbol{y}_{t-p} + \mathbf{U}\boldsymbol{\epsilon}_t(s_t)\,. \tag{3}$$

[1] A square matrix $\mathbf{U}$ is said to be unitary, if $\mathbf{U}^*\mathbf{U} = \mathbf{I}$, where an asterisk denotes a Hermitian adjoint. If $\mathbf{U}$ consists of real elements, it is called real orthogonal; see Horn and Johnson (2013).

The familiar SVAR identification issue in the case of $m = 1$ manifests itself in the observational equivalence between the reduced-form VAR model (2) and any of the (infinitely many) SVAR models in (3); see Rubio-Ramirez, Waggoner and Zha (2010). Traditional identification schemes impose restrictions on the system matrices $\mathbf{A}_0, \ldots, \mathbf{A}_p$, so that a unique SVAR model can be pinned down among a multitude of observationally equivalent ones, where the restrictions fall into one of the following categories:

- The short-run restrictions of Sims (1980), recursive schemes of Christiano, Eichenbaum and Evans (1999) and non-recursive identification of Gordon and Leeper (1994) impose direct (zero) restrictions on the elements of $\mathbf{A}_0$ or $\mathbf{A}_0^{-1}$ which are motivated by some theoretical considerations, but do not allow testing of their statistical validity;

- The long-run identification schemes of Blanchard and Quah (1989) restrict certain combinations of $\mathbf{A}_0$ and the autoregressive parameters $\mathbf{A}_1$, $\ldots, \mathbf{A}_p$, but this approach is often insufficient to draw out statistical information about the shocks of interest and cannot be statistically verified;

- The sign restrictions of Canova and De Nicolo (2002) and Uhlig (2005) use theory-based assumptions on short-run responses to certain shocks, leaving the underlying SVAR model essentially unidentified; see Rubio-Ramirez et al. (2005).

In contrast to the traditional schemes above, the burden of SVAR identification in this paper is shifted from the restrictions on system matrices $\mathbf{A}_0, \ldots, \mathbf{A}_p$ to certain conditions on the number and the uniqueness of the volatility states in (1). Consider model (3) when $m > 1$; for each $s \in \{1, \ldots, m\}$, the variance-covariance matrix of structural innovations is given by:

$$\mathbb{E}\, \mathbf{U}\boldsymbol{\epsilon}_t(s)\boldsymbol{\epsilon}_t(s)'\mathbf{U}' = \mathbf{U}\mathbf{D}(s)\mathbf{U}' .$$

In general, the right hand side matrix in this expression differs from $\mathbf{D}(s)$ for $s > 1$, whereby the model identification is achieved by choosing $\{\mathbf{D}(s) : 1 \leq s \leq m\}$ such that the variance-covariance structure of innovations in (3) is preserved only for a trivial set of unitary matrices $\mathbf{U}$.[2] A sufficient condition on $\{\mathbf{D}(s) : 1 \leq s \leq m\}$ that identifies the model through volatility in this way is given in Proposition 1 of Lanne et al. (2010), requiring each pair of structural innovations in (1) to have distinct variances in at least one of the volatility

[2]For example, $\mathbf{U}$ consisting of plus or minus ones on the main diagonal will preserve the variance-covariance structure of the innovations in (3). But this corresponds to the case of flipping the shock signs and presents just a normalization issue, since the economic impact of structural innovations remains unchanged.

states in the model. In other words, we pin down a unique SVAR model using assumptions on $\{\mathbf{D}(s) : 1 \leq s \leq m\}$, leaving the system matrices $\mathbf{A}_0, \dots, \mathbf{A}_p$ free of *a priori* restrictions.[3]

From the econometric perspective, the main challenge is to recover the contemporaneous parameter matrix $\mathbf{A}_0$ from the reduced-form VAR model (2), specifically from the family of variance-covariance matrices $\{\mathbf{\Sigma}(s) : 1 \leq s \leq m\}$. A result in matrix analysis states that for any pair of Hermitian matrices $\mathbf{\Sigma}_1$ and $\mathbf{\Sigma}_2$, where at least one is positive definite,[4] there exists a non-singular matrix $\mathbf{A}$ such that both $\mathbf{A}'\mathbf{\Sigma}_1\mathbf{A}$ and $\mathbf{A}'\mathbf{\Sigma}_2\mathbf{A}$ are diagonal; see Theorem 7.6.4 in Horn and Johnson (2013). To the best of our knowledge, no mathematical results of a similar generality exist for when there are more than two matrices; in other words, without further assumptions, no diagonalizaton of a family of (positive definite) Hermitian matrices $\mathbf{\Sigma}_1, \dots, \mathbf{\Sigma}_m$ by joint $*$congruence can be achieved when $m > 2$.[5] Therefore, we limit our empirical applications in Section 4 to the case of two volatility states, where the recovery of the contemporaneous parameter matrix $\mathbf{A}_0$ is guaranteed by the existing mathematical result without any additional restrictions.[6]

In short, the MS-SVAR model (1) can be identified by imposing certain requirements on the number of volatility states, m, and the variance-covariance matrices of structural innovations $\{\mathbf{D}(s) : 1 \leq s \leq m\}$, but otherwise without any additional restrictions on the system matrices $\mathbf{A}_0, \dots, \mathbf{A}_p$. This offers a crucial advantage over the usual identification schemes, where the shape and nature of the orthogonalized impulse response functions may be altered in the identification process. The alternating volatility regimes of the structural innovations, on the other hand, enable us to draw out a full and unrestricted contemporaneous parameter matrix $\mathbf{A}_0$ together with its sibling — the short-run impact matrix $\mathbf{A}_0^{-1}$, and to disentangle different shocks by looking at the estimated responses of system variables and suitable economic narratives without imposing *a priori* assumptions on how the shocks ought to affect the system dynamics.

[3]Although a unique SVAR model is statistically identified, the resulting shocks lack an economic interpretation. A suitable narrative for the identified shocks ought to be found by a careful analysis of the estimated short-run impacts and impulse responses, and consultation of the relevant theoretical and empirical literature. This procedure is similar to the one used by Lütkepohl and Netšunajev (2013) for checking sign restrictions, and is carried out in Section 4.

[4]In full generality, it is sufficient that a real linear combination of $\mathbf{\Sigma}_1$ and $\mathbf{\Sigma}_2$ is positive definite.

[5]Two square matrices $\mathbf{B}$ and $\mathbf{C}$ are said to be $*$congruent, if there exists a non-singular matrix $\mathbf{A}$ such that $\mathbf{A}^*\mathbf{B}\mathbf{A} = \mathbf{C}$; see Horn and Johnson (2013).

[6]Without doubt, the case with more than two volatility states is interesting from both the applied and theoretical perspectives. Such an extension, however, does not appear to be trivial in the Bayesian context.

3. Statistical inference

We use a Bayesian approach to obtain statistical inference on the relevant model parameters in the MS-SVAR model (1). It is well known that the posterior inference based on the full likelihood function in hidden Markov models is complex and computationally expensive; see Marin, Mengersen and Robert (2005). Among several proposed solutions to this issue, the Gibbs sampler combined with data augmentation is the most popular in the applied literature. In this paper we use the Gibbs sampler in the context of a hidden Markov model for the volatility of innovations; see Carter and Kohn (1994), Chib (1996) and Krolzig (1997).

Assume, in the context of the MS-SVAR model (1), that the hidden volatility state evolution is given by:

$$s_t \,|\, s_{t-1} \sim \text{Markov}(\,\mathbf{P}, \boldsymbol{\eta}_0\,)\,,$$

where the $m \times m$ matrix $\mathbf{P}$ governs the conditional distribution of state transitions, and s_0 is distributed according to the m-dimensional vector $\boldsymbol{\eta}_0$. The trajectory of hidden states $S_T := \{s_1, \ldots, s_T\}$ is obtained by simulation, where T denotes the sample size, and conditional on this, the reduced-form variance-covariance matrices $\{\boldsymbol{\Sigma}(s) : 1 \leq s \leq m\}$ can be estimated using the VAR model residuals split across different volatility states. Bayesian inference on the remaining parameters $\boldsymbol{\beta} := \text{vec}(\boldsymbol{c}_0, \boldsymbol{c}_1, \boldsymbol{\Phi}_1, \ldots, \boldsymbol{\Phi}_p)$ is similar to the usual GLS estimator of the linear regression model with heteroscedastic innovations; see Geweke (1993) and Krolzig (1997).

More specifically, our Gibbs sampler for the MS-SVAR model includes the following four steps, repeated over the desired number of iterations:[7]

1. S_T is generated by drawing in reverse time order from the posterior distribution:
$$p(s_t \,|\, Y_T, s_{t+1}) \propto p(s_t \,|\, Y_t) \cdot p(s_{t+1} \,|\, s_t)\,, \tag{4}$$
where the first term in the expression is generated recursively using Chib (1996) Bayesian simulation algorithm for hidden Markov models. It involves the prediction:
$$p(s_t \,|\, Y_{t-1}) = \sum_{s=1}^{m} p(s_t \,|\, s_{t-1} = s) \cdot p(s_{t-1} = s \,|\, Y_{t-1})\,,$$

[7] As usual in the recursive Bayesian simulation algorithms, each step of the Gibbs sampler depends partially on the previous iterative draw; see Robert and Casella (2004). We economize on the notation by only showing dependence on Gibbs draws within the same sampler iteration. In addition, all Gibbs sampler expressions in this section are conditioned on the "pre-sample" observations $\boldsymbol{y}_0, \ldots, \boldsymbol{y}_{1-p}$.

and update steps:

$$p(s_t \,|\, Y_t) \propto p(s_t \,|\, Y_{t-1}) \cdot \ell(\boldsymbol{y}_t \,|\, Y_{t-1};\, \boldsymbol{\beta}, \boldsymbol{\Sigma}(s_t))\,,$$

where Y_t denotes sample data up to $1 \leq t \leq T$, and $\ell(\boldsymbol{y}_t \,|\, Y_{t-1}; \boldsymbol{\beta}, \boldsymbol{\Sigma}(s_t))$ is the Gaussian likelihood function of $\boldsymbol{y}_t$ for a given volatility state $s_t \in \{1, \ldots, m\}$;

2. Given a simulated trajectory S_T from the previous step, the Markov transition kernel $\mathbf{P}$ is updated element-by-element, where for each $s \in \{1, \ldots, m\}$ the posterior probability of leaving the volatility state s is given by the following discrete distribution:[8]

$$p_s \,|\, Y_T, S_T \sim \text{Dirichlet}(\, \alpha_{s1} + n_{s1}(S_T), \ldots, \alpha_{sm} + n_{sm}(S_T)\,)\,, \qquad (5)$$

where $\{\alpha_{sk} : 1 \leq s, k \leq m\}$ are hyper-parameters of the Dirichlet prior for $\mathbf{P}$, and $n_{sk}(S_T)$ is the number of transitions from state s to state k in the given trajectory S_T;

3. The posterior distributions of the reduced-form variance-covariance matrices for each state $s \in \{1, \ldots, m\}$ are given by:

$$\boldsymbol{\Sigma}^{-1}(s) \,|\, Y_T, S_T \sim \text{Wishart}(\, [\mathbf{C}(s) + \bar{\mathbf{C}}(s)]^{-1};\, \tau(s) + T(s)\,)\,, \qquad (6)$$

where the family of $n \times n$ non-singular matrices $\{\mathbf{C}(s) : 1 \leq s \leq m\}$ and scalars $\{\tau(s) : 1 \leq s \leq m\}$ are hyper-parameters of the Wishart priors for $\{\boldsymbol{\Sigma}(s) : 1 \leq s \leq m\}$, and $\bar{\mathbf{C}}(s)$ are estimated variance-covariance matrices of model residuals belonging to a particular volatility state s, and $0 \leq T(s) \leq T$ is the number of occurrences of s in S_T:

$$\bar{\mathbf{C}}(s) := \sum_{t=1}^{T} \bar{\boldsymbol{u}}_t(\boldsymbol{\beta})\, \bar{\boldsymbol{u}}_t'(\boldsymbol{\beta}) \cdot \mathbf{1}\{s_t = s\}\,, \quad T(s) := \sum_{t=1}^{T} \mathbf{1}\{s_t = s\}\,,$$

$$\bar{\boldsymbol{u}}_t(\boldsymbol{\beta}) := \boldsymbol{y}_t - \boldsymbol{c}_0 - \boldsymbol{c}_1 t - \boldsymbol{\Phi}_1 \boldsymbol{y}_{t-1} - \ldots - \boldsymbol{\Phi}_p \boldsymbol{y}_{t-p}\,;$$

4. The posterior distribution of the reduced-form VAR coefficients is Gaussian:

$$\boldsymbol{\beta} \,|\, Y_T, S_T, \{\boldsymbol{\Sigma}(s) : 1 \leq s \leq m\} \sim \text{Normal}(\, \bar{\boldsymbol{b}}, \bar{\mathbf{B}}\,)\,, \qquad (7)$$

where parameters of this distribution are given by the usual GLS expressions:

$$\bar{\boldsymbol{b}} = \bar{\mathbf{B}}\, (\mathbf{X}' \otimes \mathbf{I}_n)\, \boldsymbol{\Omega}(S_T)\, \boldsymbol{y}\,, \qquad \bar{\mathbf{B}} = \left[(\mathbf{X}' \otimes \mathbf{I}_n)\, \boldsymbol{\Omega}(S_T)\, (\mathbf{X} \otimes \mathbf{I}_n) \right]^{-1},$$

[8] For mathematical details on Dirichlet and Wishart distributions see Poirier (1995).

where the $nT \times nT$ block-diagonal matrix $\mathbf{\Omega}(S_T)$ is defined as follows:

$$\mathbf{\Omega}(S_T) := \begin{pmatrix} \mathbf{\Sigma}^{-1}(s_1) & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \mathbf{\Sigma}^{-1}(s_T) \end{pmatrix},$$

where $\boldsymbol{y} := (\boldsymbol{y}_1', \dots, \boldsymbol{y}_T')'$ is a $nT \times 1$ data vector, and each row of a $T \times (2 + np)$ data matrix $\mathbf{X}$ contains the following elements:

$$(1, t, \boldsymbol{y}_{t-1}', \dots, \boldsymbol{y}_{t-p}') .$$

The most direct and computationally economical way of incorporating normal informative priors about $\boldsymbol{\beta}$ into (7) is to use the dummy variable approach of Theil and Goldberger (1961).

A recursive iteration on (4) to (7) produces, after a pre-specified number of burn-in steps, a sequence of posterior draws of S_T, $\boldsymbol{\beta}$, and the variance-covariance matrices $\{\mathbf{\Sigma}(s) : 1 \leq s \leq m\}$. Posterior sampling of the contemporaneous parameter matrix $\mathbf{A}_0$ of the MS-SVAR model (1) relies on the previously cited matrix decomposition result in the case of two volatility states:[9]

$$\mathbf{\Sigma}^{-1}(1) = \mathbf{A}_0'\mathbf{A}_0 , \quad \mathbf{\Sigma}^{-1}(2) = \mathbf{A}_0'\,\mathbf{D}(2)^{-1}\mathbf{A}_0 ,$$

which is guaranteed to exist for any two positive definite Hermitian matrices $\mathbf{\Sigma}^{-1}(1)$ and $\mathbf{\Sigma}^{-1}(2)$ produced by the Gibbs sampler, and is unique up to the column signs of $\mathbf{A}_0$ when the identification conditions of Lanne et al. (2010) are satisfied. We also note that Lanne et al. (2010) conditions are required not just for the MS-SVAR identification and posterior inference on $\mathbf{A}_0$, but also to provide a sufficient degree of separation between the volatility states and the achievement of reliable simulations of the S_T trajectories using the Chib (1996) algorithm.

Given posterior simulations of $\mathbf{A}_0$ together with reduced-form VAR matrices $\mathbf{\Phi}_1, \dots, \mathbf{\Phi}_p$, posterior impulse responses can be drawn in the usual manner; see Hamilton (1994) and Lütkepohl (2005). In all empirical applications in Section 4 we show median responses together with the corresponding pointwise 68% posterior credible sets.

[9]The proof of Theorem 7.6.4 (a) on page 487 of Horn and Johnson (2013) can serve as a template for the computer implementation of this matrix decomposition result.

4. Empirical applications

4.1. The US monetary policy

Stock and Watson (2002), Primiceri (2005), Sims and Zha (2006), Justiniano and Primiceri (2008) and many others note that US macroeconomic dynamics since the mid-1960s have been characterized by a time-varying volatility of shocks, while the remaining reduced-form VAR parameters can be considered time-invariant. This view is consistent with what is called the "good luck" explanation of the Great Moderation since the mid-1980s.[10] In this paper we use this empirical regularity of the US data for the statistical identification of shocks using the MS-SVAR framework outlined in Section 2.

The US macroeconomic data are quarterly and seasonally adjusted, covering the time period from 1964Q2 to 2009Q4. The data are supplied by the Federal Reserve Bank of St. Louis FRED database.[11] Per capita aggregates are computed using the US civilian non-institutional population aged from 16 years up. Figure 1 shows the data:[12]

- Output growth rate (Δy_t) is defined as scaled quarter-on-quarter log real GDP per capita changes;
- Inflation rate (π_t) is defined as scaled quarter-on-quarter log changes of personal consumption expenditures core price index;
- Real money balances (M_t) is defined as the sum of de-trended log inverse money velocity[13] and log real output per capita;
- Monetary policy interest rate (r_t) is defined as the average quarterly federal funds rate.

Using these data, we have estimated a MS-SVAR model with three autoregressive lags and two volatility states using the Gibbs sampler set out in

[10] A comprehensive account of the Great Moderation and an up-to-date literature survey on the topic can be found in Davis and Kahn (2008).

[11] All data series are downloaded from *research.stlouisfed.org/fred2*

[12] As a robustness check, in all our applications in this section we have used de-trended output in place of the output growth rates as an alternative business cycle measure, and a GDP deflator-derived inflation measure as a substitute for headline consumer inflation. Our main empirical results and conclusions remain unchanged.

[13] Inverse money velocity is calculated as a ratio of the quarterly sweep-adjusted M2 money stock and quarterly nominal GDP; see Cynamon et al. (2006).

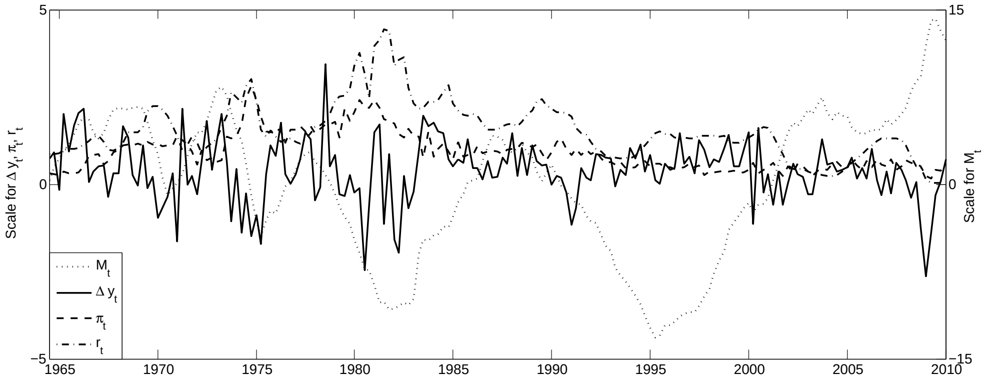


Figure 1: The US macroeconomic data series

Section 3.[14] The augmentation choice is a compromise between the two lags preferred by the Bayesian information criteria and the five lags favored by the Akaike information criteria, and it takes into account the fact that real output enters the model in the log-differenced form. Our findings are robust with respect to the augmentation, with no substantial changes in reported results for two or four autoregressive lags in the empirical model.

The following priors are used in the model estimated for the US data: the hyper-parameters of the Dirichlet prior on the elements of the Markov transition kernel $\mathbf{P}$ are $\alpha_{sk} = 10$ for $s = k$ and $\alpha_{sk} = 1$ for $s \neq k$, where $s, k \in \{1, 2\}$. The Wishart prior hyper-parameters on the reduced-form variance-covariance matrices $\mathbf{\Sigma}(1)$ and $\mathbf{\Sigma}(2)$ are given by $\mathbf{C}(1) = 0.350 \cdot \mathbf{I}$, $\mathbf{C}(2) = 0.175 \cdot \mathbf{I}$, $\tau(1) = 12$, and $\tau(2) = 6$. Non-informative priors on $\boldsymbol{\beta}$ are used for the remaining parameters of the model. The prior selection is primarily focused on achieving a reliable separation between the two volatility states in the US sample, helping to avoid the label-switching issue that often plagues empirical applications of hidden Markov models. At the same time, the flat $\boldsymbol{\beta}$ priors impose minimum *a priori* conditions on the economically important parts of the model.

The average of the simulated S_T trajectories is shown in Figure 2. State 1 is the high volatility state, which can clearly be associated with the periods of economic downturn and uncertainty following the first and the second oil crises in the early 1970s and 1980s, and the 2007–2009 Global Financial Crisis. State 2 is the low volatility state, capturing several short tranquil periods in 1960s, mid-1970s, and the long Great Moderation period from the mid-1980s

[14]In all empirical applications in this section we run the Gibbs sampler loop for 2000 iterations, starting from a suitable prior distribution draw, and use the last 500 simulations for calculating posterior statistics. All the computations in this paper are carried out in the Ox matrix programming language; see Doornik (2007).

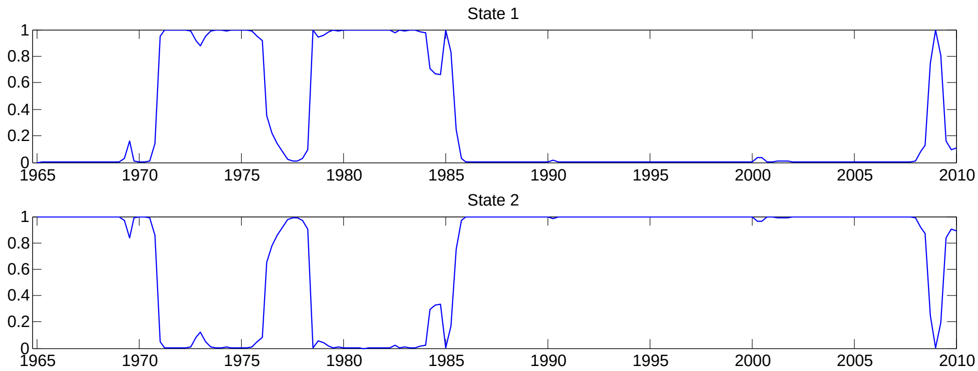


Figure 2: Average of simulated S_T trajectories for the US data

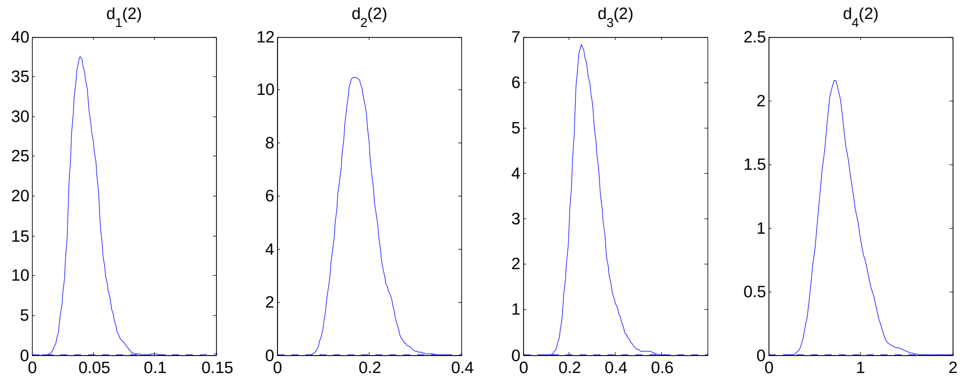


Figure 3: Prior (dashed line) and posterior (solid line) distributions of $\mathbf{D}(2)$ for the US data

until the outbreak of the 2007–2009 Global Financial Crisis. The two volatility states are prominent in the US sample and are well in line with the profession's consensus on US macroeconomic history over the last five decades; see Stock and Watson (2002) and Sims and Zha (2006). Economic history aside, from the perspective of this paper, the well pronounced volatility regimes in the US data should allow us to draw out a complete set of statistically identified and meaningful macroeconomic shocks in our model.

The condition for local identification of $\mathbf{A}_0$ in a two-state MS-SVAR model requires all the diagonal elements of $\mathbf{D}(2)$, henceforth denoted as $d_1(2), \ldots, d_n(2)$, to be distinct; see Section 2 and Lanne et al. (2010). The priors and posteriors of $d_1(2), \ldots, d_4(2)$ for the US data are shown in Figure 3. While the data are clearly informative about $d_1(2), \ldots, d_4(2)$, we additionally compute

Table 1: 68% and 90% posterior credible sets of $\mathbf{D}(2)$ for the US data

$\mathbf{D}(2)$	68% set	90% set
$d_1(2)$	$[0.0245, 0.0645]$	[0.0194, 0.0751]
$d_2(2)$	$[0.1067, 0.2518]$	[0.0921, 0.2764]
$d_3(2)$	$[0.1735, 0.4048]$	[0.1515, 0.4695]
$d_4(2)$	$[0.4300, 1.1792]$	[0.3616, 1.3095]

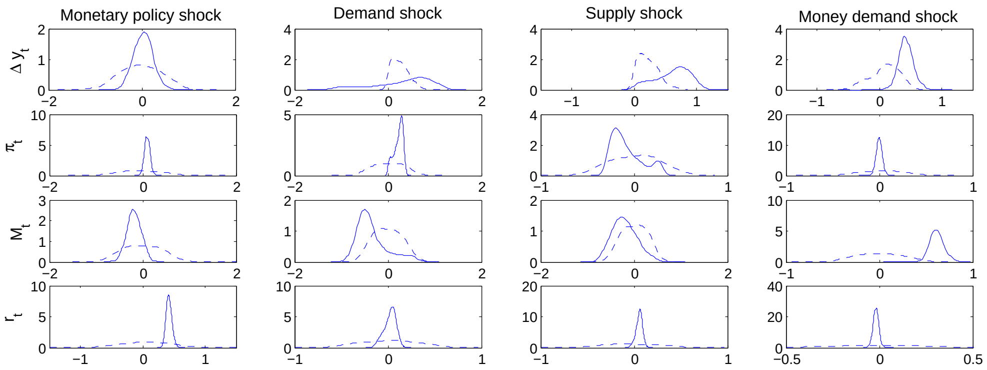
68% and 90% posterior credible sets in order to check whether the diagonal elements of $\mathbf{D}(2)$ are distinct from each other at the given levels. Computed posterior credible sets are shown in Table 1, from where it follows that $d_1(2)$ and $d_4(2)$ are well separated in both cases. On the other hand, the posterior distributions of $d_2(2)$ and $d_3(2)$ elements partly overlap in the 68% and 90% level cases, and the 90% level sets of $d_3(2)$ and $d_4(2)$ also appear to intersect. This has strong implications for the posterior short-run effects and impulse responses: while the first and fourth shocks are likely to be well identified in the estimated model, it may be more difficult to isolate and attach economic interpretations to the second and third shocks in the US data model.

It should be recalled that the contemporaneous parameter matrix $\mathbf{A}_0$ and, by extension, the short-run impact matrix $\mathbf{A}_0^{-1}$ are identified only up to the signs of their rows or columns; see Section 2. In the application to the US data we normalize the short-run impact matrix such that the initial effects of the first shock on r_t (column one of $\mathbf{A}_0^{-1}$), of the second shock on π_t (column two of $\mathbf{A}_0^{-1}$), of the third shock on Δy_t (column three of $\mathbf{A}_0^{-1}$), and of the fourth shock on M_t (column four of $\mathbf{A}_0^{-1}$) are all positive. The prior and posterior distributions of the elements of the short-run impact matrix $\mathbf{A}_0^{-1}$ are shown in Figure 4. Again, we find the US data reasonably informative about $\mathbf{A}_0^{-1}$, especially in the first and the last columns of the matrix. We also observe that the posterior distributions of the two middle columns tend to be wider and in some cases exhibit bi-modality; this is likely to be a consequence of insufficient separation between the $d_2(2)$ and $d_3(2)$ diagonal elements of $\mathbf{D}(2)$ and the resulting identification difficulties.

By examining the last row of $\mathbf{A}_0^{-1}$ in Figure 4, we can identify a "monetary policy shock" in the estimated system: r_t shows the strongest immediate reaction to the first shock, while in the other columns the biggest initial reactions are associated with variables other than r_t.[15] The first shock increases the

[15]Since the identification methodology in this paper is statistical and does not impose *a priori* economic meaning to different shocks, it is necessary to examine meticulously all the available statistical evidence before attaching interpretations to particular shocks. The inter-

Figure 4: Prior (dashed line) and posterior (solid line) distributions of $\mathbf{A}_0^{-1}$ for the US data



policy rate as a result of normalization, with an estimated immediate positive impact on prices and a negative one on real money balances, while the instantaneous reaction of real output remains ambiguous. In terms of the short-run impact, our "monetary policy shock" is close to its sibling in Uhlig (2005), apart from the opposite reaction of π_t. More specifically, we also find that the initial real output response is ambiguous and the money supply tightens on impact. The opposite reaction of prices, on the other hand, is likely to be explained by the imposed identification assumption in Uhlig (2005), while in our case the positive initial reaction of π_t appears to be a feature of the US data delivered by our identification methodology. Importantly, the estimated posterior distribution of the immediate π_t reaction to our "monetary policy shock" in Figure 4 is predominantly concentrated on the positive half-line, which has strong implications for the sign identification approach of Uhlig (2005): while the negative initial effects on π_t and M_t are within the domain of our posterior, the probability of this reaction mix appears to be quite low.[16]

Similar discrepancies in terms of the short-run monetary policy impact on π_t are observed between the traditional recursive and non-recursive identification schemes of Christiano et al. (1999) and Gordon and Leeper (1994) and

pretation of shocks in estimated MS-SVAR models should be based on their short-run impact, posterior impulse responses and theoretical considerations, all taken together and carefully weighted. On its own, the strongest initial reaction of r_t to the first shock may not be sufficient to justify its labeling; our interpretation of this shock also stems from the posterior impulse responses, which are detailed later in this section.

[16]In a tri-variate monetary SVAR, estimated without real money and identified using sign restrictions, Castelnuovo and Surico (2010) relax the assumption of the non-positive response of prices to a monetary policy shock and find strong evidence in favour of the "price puzzle". However, they claim that this finding is limited to the pre-1980 US sample, before the start of the Volcker reserve targeting regime; see also Boivin and Giannoni (2006).

the results shown in Figure 4. It is common in the traditional schemes to allow the money supply to tighten on impact, while assuming a reaction lag of both real output and prices to a contractionary monetary policy shock. Our statistical identification approach, on the other hand, yields a strong positive reaction of prices on impact and takes us back to the classical "price puzzle" result of Sims (1992). In contrast to the recent literature, Sims allows for an immediate reaction of real and nominal quantities to a monetary policy shock and finds a pronounced initial response of prices, common across several alternative datasets and model specifications. Further investigation of the strong positive initial reaction of π_t to a contractionary monetary policy shock and its implications for the traditional identification schemes is warranted.[17]

The other well-identified part of $\mathbf{A}_0^{-1}$ in Figure 4 is the last column, corresponding to the short-run impact effect of the fourth shock in the estimated system. As explained above, $d_4(2)$ is sufficiently well separated from the other three diagonal elements of $\mathbf{D}(2)$, and the posterior distributions in the last column of $\mathbf{A}_0^{-1}$ have regular shapes, distinct from the corresponding priors. The fourth shock increases real money due to the adopted normalization, having a strong positive effect on the real output, a weaker negative impact on the policy rate, and an uncertain immediate effect on prices. We label this shock a "money demand shock", partly because the nature of its immediate impact bears some resemblance to the available empirical evidence in Favara and Giordani (2009) and others, and partly due to theoretical considerations whereby the money supply is linked to the Federal Reserve policy and therefore already subsumed by the first shock in our system.

Because of the identification issues in our estimated US data model, the remaining two shocks do not appear to exhibit clear and well pronounced short-run effects on the system variables; see the two middle columns in Figure 4. We will offer a plausible interpretation of these two shocks after examining their posterior impulse responses.

The posterior impulse response functions together with the point-wise 68% credible sets, estimated for the US data, are shown in Figure 5.[18] Our preliminary shock identification exercise, based on the posterior short-run impact matrix $\mathbf{A}_0^{-1}$, yielded two provisional candidates: one for a "monetary policy

[17] An explanation of this result, suggested by Sims (1992), links the reaction of the monetary policy authority to anticipated inflationary pressures, which creates a statistical illusion of a causal relationship between monetary policy shocks and subsequent price increases. Our MS-SVAR model, estimated and presented in this subsection, is intended for statistical identification of shocks, and is not designed to test Sims' hypothesis.

[18] Although the US model in this subsection is estimated using log-differenced real per capita output, the posterior impulse response functions in the first row of Figure 5 show the accumulated (level) responses of output to different shocks. The same convention applies to the euro area and Estonian economy models in this section.

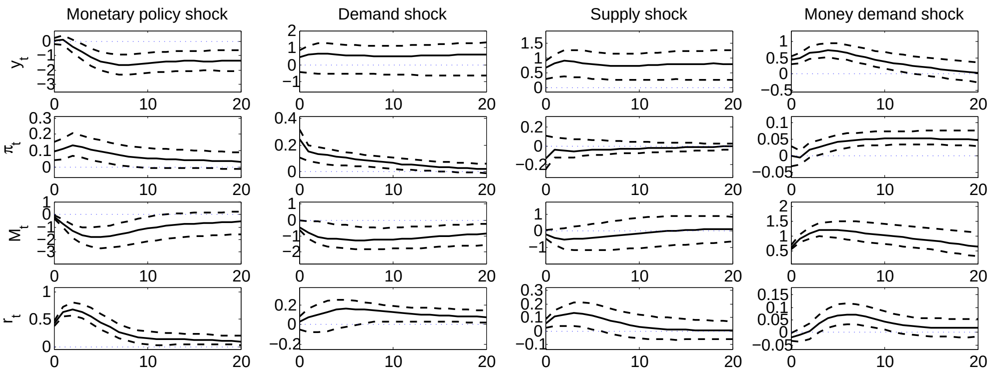


Figure 5: Posterior impulse responses with point-wise 68% credible sets for the US data

shock" (first in the system) and another for a "money demand shock" (last in the system). Looking at the estimated reactions of the model variables in the first column of Figure 5, a response profile consistent with contractionary monetary policy shocks is evident: the policy rate increase leads to a reduction in real money balances, a drop in real output and the "price puzzle" reaction of the inflation rate noted earlier.[19] This reaction would be typical for a monetary policy shock considered in the studies by Sims (1992), Christiano et al. (1999) and Sims and Zha (2006), among many others. The last column in Figure 5 depicts the reactions of the model variables to our "money demand shock": an increase in real money balances leads to a delayed reaction of the policy rate, and a strong positive and persistent response of the real output and prices. This reaction profile corroborates the recent finding in Favara and Giordani (2009) and lends further support for the hypothesis that, contrary to the standard New Keynesian framework, money demand shocks, which may be linked to widening of term spreads, increasing stock returns or exchange rate deprecations, lead to well-pronounced responses of real macroeconomic aggregates.[20]

A pair of hitherto unidentified shocks present a challenge for interpreting

[19]The "price puzzle" in our posterior impulse responses is robust to alternative model specifications. In particular, it remains virtually unchanged if we replace real output growth with de-trended output, as suggested in Giordani (2004). Adding a global commodity price index to the list of model variables makes the initial response of inflation to our "monetary policy shock" smaller, so its 68% posterior credible set now contains zero, but it still remains sufficiently large for the subsequent two to four quarters to be taken as a credible evidence in favour of the "price puzzle".

[20]Andres, Lopez-Salido and Nelson (2009) re-visit the role of money in the New Keynesian framework, finding that certain variations of these structural macroeconomic models imply a forward-looking character of the real money balances, helping to predict future variations in the natural interest rate and other real aggregates.

the results when our identification methodology fails to deliver clear-cut short-run impacts and impulse responses due to the insufficient statistical information in the volatility regimes. In particular, the posterior impulse responses in the two middle columns of Figure 5 are not sharp enough for us to discern confidently the statistical effects of the two shocks on real output and prices. However, judging by the median responses of y_t and π_t, the second column is consistent with a positive "aggregate demand shock", pushing output and prices in the same direction and inducing a pronounced reaction of the monetary authority after an initial delay. Using similar arguments, we identify the third shock in our systems as a positive "aggregate supply shock", where the median reactions of output and inflation have the opposite signs. Our estimates suggest that the Federal Reserve may react to the aggregate supply shock by increasing the policy rate, although the identification issues in our model cast some doubt on the statistical reliability of this finding.

In summary, we apply the four-variate two-state MS-SVAR model from Section 2 to the US macroeconomic series, uncovering a convincing conciliation of high and low-volatility states over the last half century of data. The switching volatility regimes allow us to draw out short-run impacts and impulse responses of four structural shocks, which we label a "monetary policy shock", a "money demand shock", an "aggregate demand shock" and an "aggregate supply shock". The statistical identification of the first two shocks is found to be reliable and delivers empirical results that appear to corroborate many previous findings on monetary policy and money shocks. In particular, we report a robust and well-pronounced "price puzzle" in response to our "monetary policy shock", while our "money demand shock" induces a strong response of real output and prices in the US data. While the statistical identification of the other two shocks in our system is less sharp, the estimated impulse response profiles provide enough evidence to justify their labeling as "aggregate supply" and "aggregate demand" shocks. Finally, we point out that the results of our statistical shock identification procedure are not always compatible with the traditional short-run and sign identification schemes used in much of the recent literature on SVAR models.

4.2. The euro area monetary policy

The empirical evidence on the Great Moderation and the time-varying volatility of macroeconomic shocks in the EA is more muted, given the number of institutional changes in Europe in the post-war period. Using a retrospective dataset on the core EA countries, the European Commission documents a picture of the Great Moderation in the EA similar to that in the US since the early 1970s; see Cabanillas and Ruscher (2008). Rubio-Ramirez et al. (2005) find

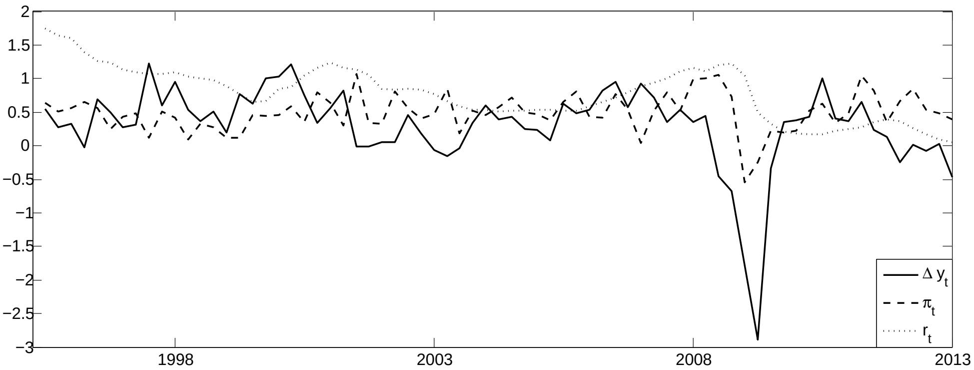


Figure 6: EA macroeconomic data series

that a reduced-form VAR model with switching volatility of shocks and time-invariant conditional mean parameters provides the best fit to the retrospective EA macroeconomic data dating from the early 1970s onwards. As in the previous subsection, we use this empirical evidence to apply our statistical shock identification method from Section 2 to a recent sample of the aggregate EA data, without imposing any *a priori* identifying restrictions.

The EA macroeconomic data is quarterly, covering the time period from 1995Q2 to 2012Q4.[21] Per capita aggregates are computed using the working age population, defined as residents from 16 to 65 years of age.[22] All series are computed in the same way as for the US data in Subsection 4.1 and displayed in Figure 6:

- Output growth rate (Δy_t) is defined as scaled quarter-on-quarter log real GDP per capita changes;
- Inflation rate (π_t) is defined as scaled quarter-on-quarter log changes of HICP index;
- Monetary policy rate (r_t) is taken to be equal to the three month EURIBOR interest rate.

Like in the previous subsection, we have estimated a two-state MS-SVAR model with three autoregressive lags. The augmentation is selected taking into account that real output enters the model in log differences. The usual

[21]The data are sourced from the European Commission's Eurostat homepage *epp.eurostat.ec.europa.eu*

[22]Retrospective population data and projections are taken from the European Commission's AMECO database at *ec.europa.eu/economy_finance/ameco*

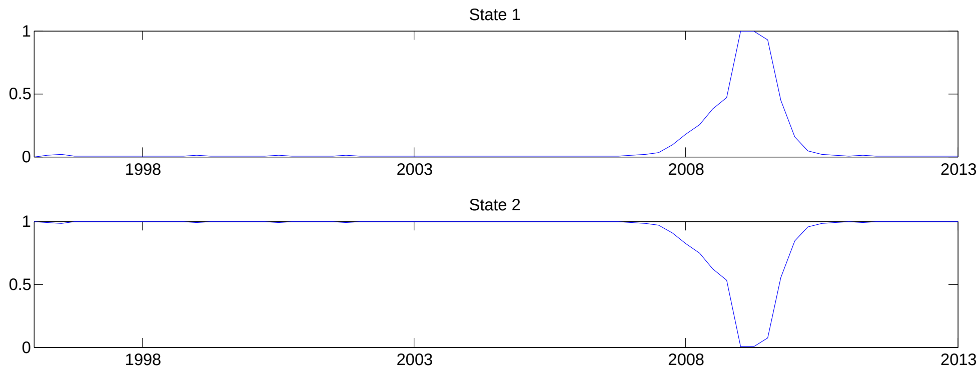


Figure 7: Average of simulated S_T trajectories for the EA data

information criteria suggest the lag order of two (using the Akaike Information Criteria) or one (using the Bayesian Information Criteria), but this may not be enough to capture fully the time series dynamics in the data. However, the results in this subsection are not sensitive to the reduced augmentation and continue to hold in the model with two autoregressive lags. Priors on the EA model parameters are similar to those in Subsection 4.1, apart from the slightly modified Wishart prior hyper-parameters on $\mathbf{\Sigma}(1)$ and $\mathbf{\Sigma}(2)$, given by $\mathbf{C}(1) = 0.310 \cdot \mathbf{I}$, $\mathbf{C}(2) = 0.155 \cdot \mathbf{I}$, $\tau(1) = 12$, and $\tau(2) = 6$.

The average of the simulated S_T trajectories is shown in Figure 7. In this figure, state 1 represents the high volatility state, clearly associated with the recent Global Financial Crisis and the subsequent European sovereign debt crisis.[23] Our estimated state probabilities are for the most part similar to the findings of Rubio-Ramirez et al. (2005), but we do not capture an elevated probability of a high volatility regime around the dot-com bubble in the start of the 2000s. In general, given that the EA sample is already small, the dearth of high volatility shocks in our model may present a problem for the statistical identification of the MS-SVAR parameters.

The prior and posterior distributions of the diagonal elements $d_1(2), \ldots,$ $d_3(2)$ of matrix $\mathbf{D}(2)$ are depicted in Figure 8. While the data are again seen to be sufficiently informative, we check the local identification of our MS-SVAR model by computing the 68% and 90% posterior credible sets of $\mathbf{D}(2)$, which are shown in Table 2. It can be seen that not all elements may be statistically distinct, as the intervals of $d_1(2)$ and $d_2(2)$ mutually overlap in the 68% and 90% cases, and the same holds true for $d_2(2)$ and $d_3(2)$. However,

[23]Note that the posterior median of $d_3(2)$ in Figure 8 is close to unity: the variance of this shock is therefore similar across the two states. However, state 1 is still a "high volatility state" in our interpretation owing to the elevated volatility of the other two shocks.

Table 2: 68% and 90% posterior credible sets of $\mathbf{D}(2)$ for the EA data

$\mathbf{D}(2)$	68% set	90% set
$d_1(2)$	$[0.0135, 0.1450]$	[0.0133, 0.1916]
$d_2(2)$	$[0.0618, 0.3661]$	[0.0423, 0.5490]
$d_3(2)$	$[0.2240, 2.2251]$	[0.1806, 2.8247]

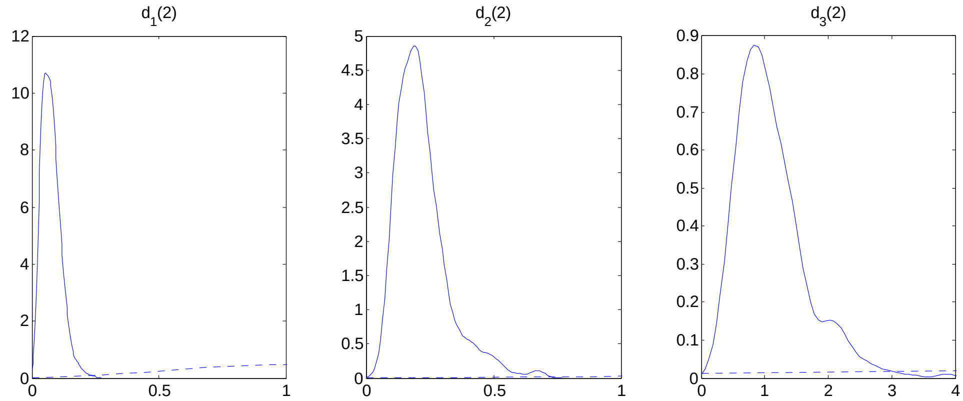


Figure 8: Prior (dashed line) and posterior (solid line) distributions of $\mathbf{D}(2)$ for the EA data

$d_1(2)$ and $d_3(2)$ appear to be sufficiently far apart in the 68% case. This is a clear indication that there may not be enough information in the estimated volatility regimes to identify all the parameters and disentangle all the shocks in the EA data. Bearing this in mind, we still take a closer look at the posterior short-run impacts and impulse response functions in the hope of getting a fuller view of economically interpretable shocks in the EA sample.

In the application to the EA data we normalize the short-run impact matrix $\mathbf{A}_0^{-1}$ such that the initial effects of the first shock on Δy_t (column one of $\mathbf{A}_0^{-1}$), of the second shock on r_t (column two of $\mathbf{A}_0^{-1}$), and of the third shock on π_t (column three of $\mathbf{A}_0^{-1}$) are all positive. Again, this normalization selects the strongest estimated initial effects of shocks on the corresponding model variables, providing a convenient starting point for the subsequent analysis.

The prior and posterior distributions of the elements in $\mathbf{A}_0^{-1}$ are shown in Figure 9. If the distribution of initial effects in the last row of $\mathbf{A}_0^{-1}$ are examined, a "monetary policy shock" candidate can be spotted: only the second shock pushes the policy rate up on impact, while the other two shocks do not appear to sway r_t in any particular direction in the first instance. In addition to this, the estimated posterior short-run impacts in our EA model on Figure 9

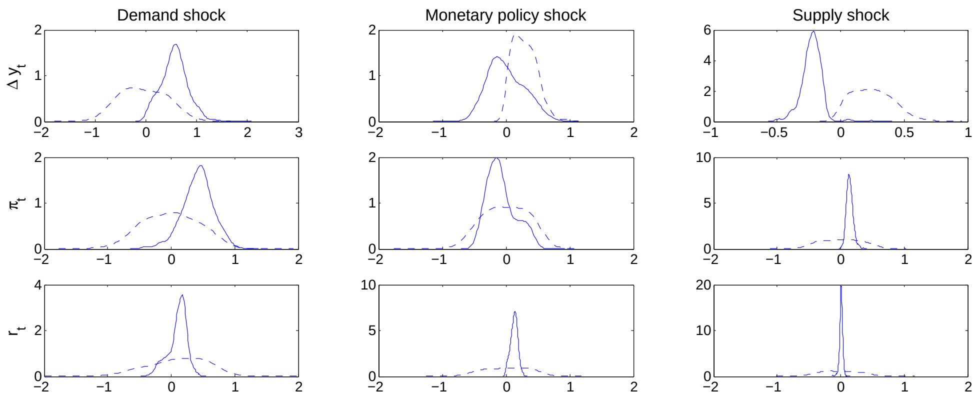


Figure 9: Prior (dashed line) and posterior (solid line) distributions of $\mathbf{A}_0^{-1}$ for the EA data

allow us to discern an "aggregate demand shock" and an "aggregate supply shock" by their opposite effects on Δy_t when the prices are pushed up. We also note slight bi-modality of the posterior distributions in the middle column of $\mathbf{A}_0^{-1}$, which may point to potential identification issues with the second shock in our model.

The estimated posterior impulse response functions for our EA model are shown in Figure 10. Overall, the estimated response profiles in Figure 10 confirm our provisional shock labels, pinned down earlier in this subsection. From the effect of the first shock, designated previously as an "aggregate demand shock", it can be seen that both output and inflation are pushed up for a period of few quarters, while the policy rate reacts after an initial delay, seen as a counteractive monetary policy measure by the European Central Bank to demand pressures in the EA economy. The second shock, corresponding to our discretionary "monetary policy shock", pushed up the policy rate, leading to lower real output and prices, although the estimated effects are not sharp due to identification issues. Again, this reaction would be typical for a monetary policy shock, consistent with earlier empirical evidence for the EA in Peersman and Smets (2001) and Angeloni, Kashyap and Mojon (2003), and a more recent study by Weber, Gerke and Worms (2011), all obtained using the conventional identification schemes. Finally, the third shock, designated as an "aggregate supply shock" on the basis of its short-run impact, induces a strong and persistent response of real output and inflation in opposite directions, while the European Central Bank, as expected, does not counteract a supply shock using its policy instrument.

The conventional zero and sign restrictions used in the empirical monetary policy studies appear to have stronger support in the EA data than in Subsec-

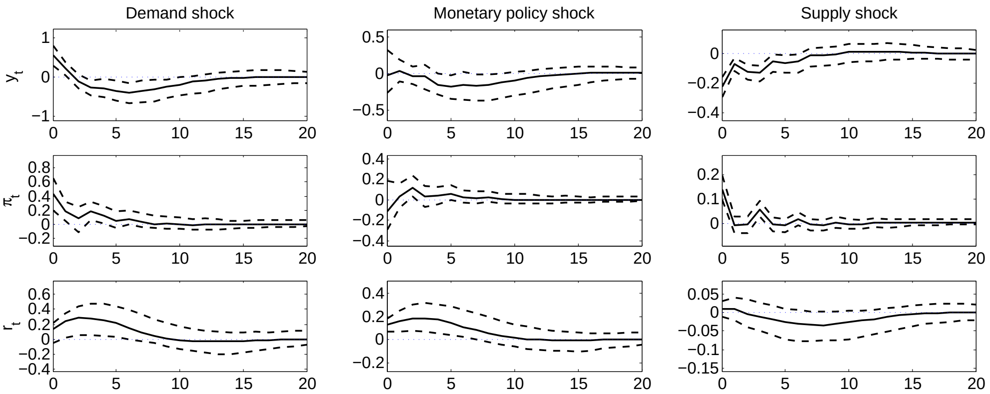


Figure 10: Posterior impulse responses with point-wise 68% credible sets for the EA data

tion 4.1. In the case of the "monetary policy shock" in our MS-SVAR model, we find that the instantaneous reactions of output and prices are close to zero, even though this result may be affected by the identification issues due to insufficient volatility information in the data. On the other hand, our findings suggest a strong initial co-movement of output and prices in the case of aggregate supply and demand shocks, which is not fully compatible with the usual recursive identification schemes.

In summary, an application of our statistical shock identification methodology to the EA data delivers a full set of discernible structural shocks, which we label an "aggregate demand shock", an "aggregate supply shock" and a discretionary "monetary policy shock". As in the case of the US data in Subsection 4.1, the estimated posterior reactions of the EA macroeconomic aggregates to the identified shocks are mostly in line with the empirical literature, although we point out that the conventional SVAR identification scheme may not be fully compatible with the data. However, some of our findings in this subsection may be affected by the scarce volatility regimes in the EA data sample.

4.3. Response of the Estonian economy to an exogenous risk premium shock

In the final application of our statistical shock identification methodology, laid out in Section 2, we investigate the response of the main Estonian macroeconomic aggregates to an exogenous interest rate (risk) premium shock. To this end, we consider a small-scale trivariate MS-SVAR model with two volatility states, resembling the one in Subsection 4.2. Because the Estonian economy is a part of the EA, with an integrated financial system and a business cycle strongly linked to the common EA monetary policy, we assume that the policy actions of the European Central Bank transmit fully to the Estonian economy. Over and above the EA short term rates we assume the existence of an exogenous country-specific interest rate premium, manifesting itself as a spread between the Estonian and EA short-term rates, in the spirit of an incomplete market model with modified UIP condition, as in Schmitt-Grohe and Uribe (2003), Gelain and Kulikov (2011) and others.[24] However, we do not make any structural assumptions about the dynamics of the exogenous risk premium, instead relying on the in-sample volatility regimes and our MS-SVAR methodology to reveal the responses of model variables to the identified structural shocks.

The Estonian (EE) macroeconomic data are quarterly, covering the time period from 1995Q2 to 2012Q4, and are obtained from the same database and pre-processed in a identical way to the EA series in Subsection 4.2. Figure 11 depicts the following three Estonian variables:

- Output growth rate (Δy_t) is defined as scaled quarter-on-quarter log real GDP per capita changes;
- Inflation rate (π_t) is defined as scaled quarter-on-quarter log changes of the HICP index;
- Risk premium (ω_t) is defined as the spread between the three month Estonian interbank rate and the corresponding EURIBOR rate.[25]

As in Subsections 4.1 and 4.2, the EE model is estimated with two volatility states and three autoregressive lags. The information criteria suggest an augmentation order of four (using the Akaike Information Criteria) and one (using the Bayesian Information Criteria) for the EE sample. We use a compromise between the two information criteria, considering that the output enters the model in log differences. Again, our findings are robust to the augmentation choice, and continue to hold for models with up to four autoregressive

[24]Prior to becoming a full EA member in 2011, Estonia had a currency board-based monetary system, initially anchored to the Deutsche Mark and later to the euro, with free capital mobility and no independent monetary policy. After the domestic banking crisis in the second half of the 1990s and the ensuing domination of Scandinavian banks in the local market, the spreads between Estonian and EA interest rates started to narrow, reflecting strong economic growth and an inflow of foreign capital. During the 2007–2009 Global Financial Crisis, which saw the collapse of the property market and a severe business cycle downturn, the spreads increased again due to general macroeconomic and financial sector risks; see Dabušinskas and Randveer (2011).

[25]Reliable statistical data on the term structure of Estonian interbank rates is available only from 1996 to 2010, and the remaining parts of the series are pieced together from different data sources. Details are available from the authors on request.

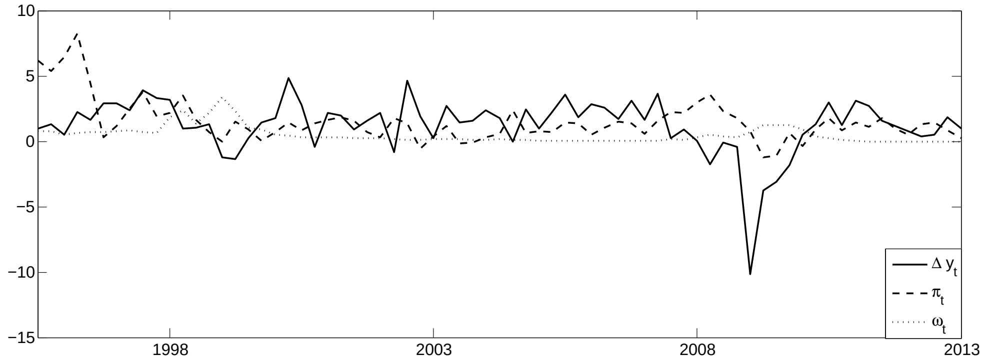


Figure 11: EE macroeconomic data series

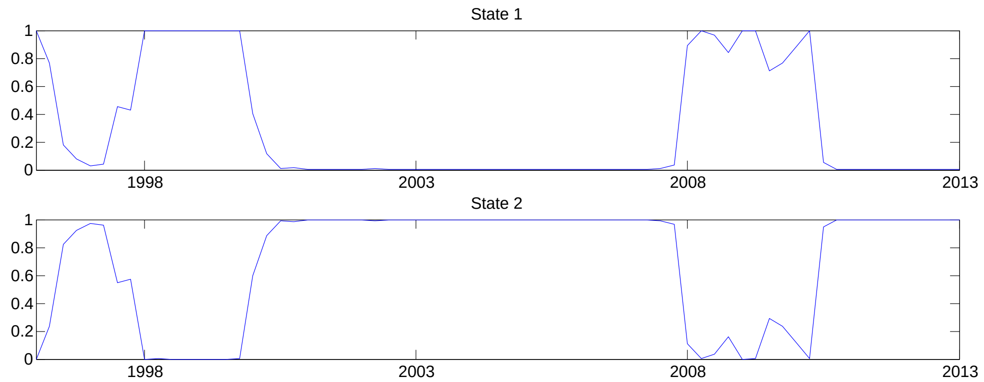


Figure 12: Average of simulated S_T trajectories for the EE data

lags. The priors are similar to those in Subsection 4.1, apart from the altered Wishart prior hyper-parameters on $\mathbf{\Sigma}(1)$ and $\mathbf{\Sigma}(2)$, which are now given by $\mathbf{C}(1) = 0.0510 \cdot \mathbf{I}$, $\mathbf{C}(2) = 0.0255 \cdot \mathbf{I}$, $\tau(1) = 10$, and $\tau(2) = 5$.

The average of the simulated S_T trajectories is shown in Figure 12. In this figure, state 1 corresponds to the high volatility state associated with some turbulent periods in Estonian macroeconomic history since the mid-1990s. The high volatility period early in the sample is linked to the Asian and Russian financial crises of 1997–1998, and the ensuing domestic banking crisis. The second highly volatile period is associated with the recent Global Financial Crisis and the resulting gyrations in the domestic business cycle. The volatility information in the EE sample appears to be richer than the EA data in Subsection 4.2, giving us a better outlook for disentangling shocks in the estimated MS-SVAR model.[26]

[26] A rather different set of dynamics of the volatility regimes in the EA and EE samples, depicted in Figures 7 and 12 respectively, frustrated our attempts to estimate a joint MS-SVAR

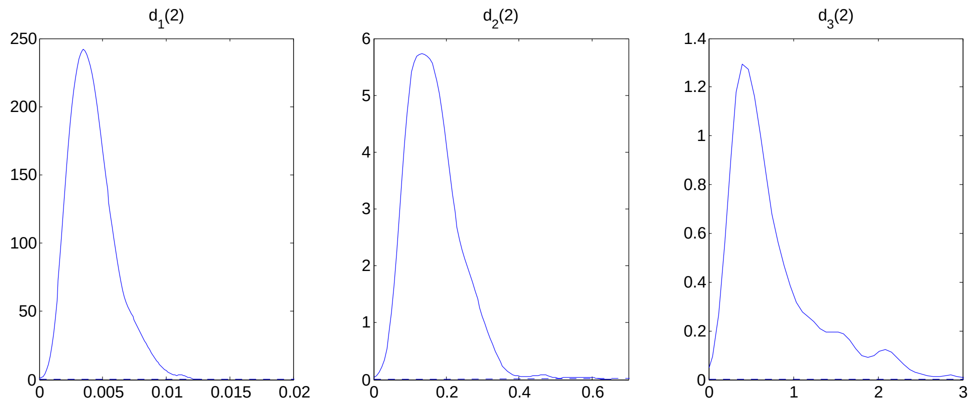


Figure 13: Prior (dashed line) and posterior (solid line) distributions of $\mathbf{D}(2)$ for the EE data

Table 3: 68% and 90% posterior credible sets of $\mathbf{D}(2)$ for the EE data

$\mathbf{D}(2)$	68% set	90% set
$d_1(2)$	$[0.0012, 0.0078]$	[0.0012, 0.0097]
$d_2(2)$	$[0.0470, 0.3052]$	[0.0279, 0.3550]
$d_3(2)$	$[0.1651, 1.7388]$	[0.1651, 2.5463]

Local identification of the two-state MS-SVAR model depends on the separation of diagonal elements of the $\mathbf{D}(2)$ matrix; the corresponding conditions have to be checked on a case-by-case basis. The prior and posterior distributions of $d_1(2), \ldots, d_3(2)$ for the EE sample are depicted in Figure 13; Table 3 shows the corresponding 68% and 90% posterior credible sets. The credible sets of $d_1(2)$ and $d_2(2)$, and of $d_1(2)$ and $d_3(2)$ do not intersect even at the 90% level, and therefore the statistical information available in the estimated volatility regimes should make it possible to disentangle at least two shocks in this application. Selecting the strongest estimated initial effects of the shocks, we normalize the short-run impact matrix as follows: the first shock has a positive initial effect on ω_t in column one of $\mathbf{A}_0^{-1}$, the second shock has a positive initial effect on Δy_t in column two of $\mathbf{A}_0^{-1}$, and the third shock has a positive initial effect on π_t in the last column of $\mathbf{A}_0^{-1}$.

The prior and estimated posterior distributions of the initial shock effects are shown in Figure 14. Only the first shock appears to push the risk premium away from zero on impact, making it our primary candidate for an exoge-

model of the EA and EE economies, which is necessary for a detailed picture of the common EA monetary policy impact on the Estonian economy to be gained.

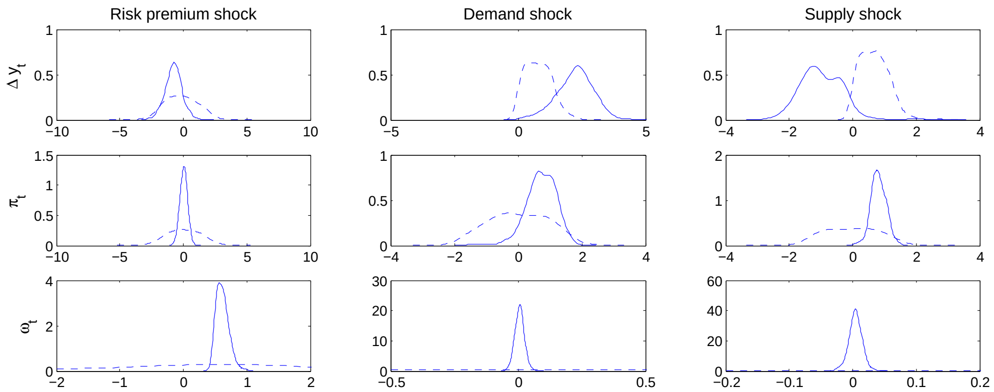


Figure 14: Prior (dashed line) and posterior (solid line) distributions of $\mathbf{A}_0^{-1}$ for the EE data

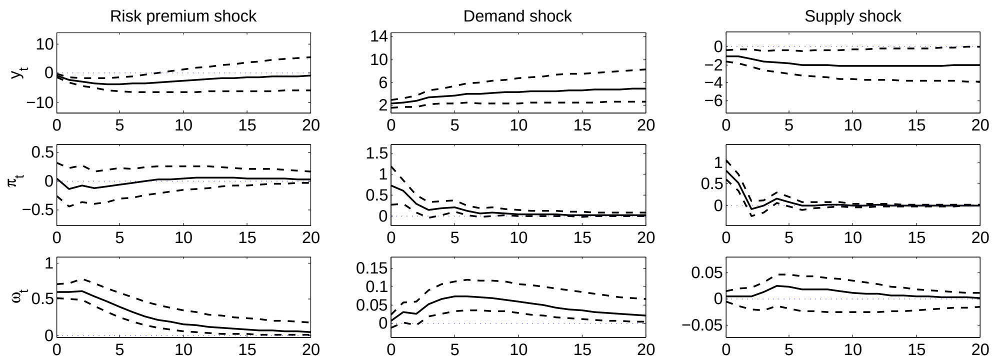


Figure 15: Posterior impulse responses with point-wise 68% credible sets for the EE data

nous "risk premium shock" in the estimated model. The next two shocks in this application affect real output and prices in a way that makes their labeling relatively straightforward: the second one pushes output and prices in the same direction, just as an "aggregate demand shock" would be expected to do, while the third one is consistent with an "aggregate supply shock" that increases prices and lowers the output in the economy. The initial risk premium reaction in the last two columns of Figure 14 is, however, ambiguous — its time dynamics, crucial for a robust economic interpretation of the model, are looked at next.

The estimated posterior impulse responses are displayed in Figure 15. The most interesting set of results in this figure comes from the endogenous reaction of the risk premium to the "aggregate demand" and "aggregate supply" shocks, and the dynamic effect of the "risk premium shock" on the rest of the model. Starting from the latter, we see that an exogenous increase in the interest rate premium (over and above the common EA monetary policy effect on the short-term rates) leads to a strong and persistent negative effect on real output, while the inflation response remains muted. Another instance where our model delivers a dynamic reaction of ω_t is associated with the "aggregate demand shock": an upturn in the Estonian economy, perhaps stemming from increased optimism among consumers and investors, or inflow of capital, induces a delayed but pronounced positive response of the risk premium, providing an endogenous counteraction similar to the debt-elastic interest rates in the Schmitt-Grohe and Uribe (2003) small open economy model. On the other hand, our empirical results indicate no reaction of ω_t in response to the "aggregate supply shock", which leads to a drop in real output due to an increased price level, owing for example to cost-push shocks or increased price mark-ups.

In summary, our estimated two-state MS-SVAR model for the Estonian economy delivers a consistent and well-defined set of shocks and dynamic responses of the main macroeconomic aggregates over the sample period 1995Q2 to 2012Q4. The rich volatility dynamics in the EE data series helped us to uncover three structural shocks, which appear to fit a theoretically sound narrative of exogenous "risk premium", "aggregate demand" and "aggregate supply" shocks. The empirical results in this subsection will prove useful in future revisions of the macroeconomic models for the Estonian economy.

5. Conclusion

SVARs are popular tools in modern macroeconomics, used as an empirical benchmark for understanding the nature of business cycles and for the development of new theories. Since their widespread adoption in the early 1980s, the on-going theoretical and empirical research has focused on the most appropriate identification strategies for different macroeconomic shocks.

In this paper we contribute to the literature on shock identification by developing a Bayesian estimator for the MS-SVAR model with time-varying volatility of structural innovations that depend on a hidden Markov process. With sufficient statistical information in the data, distinct volatility states of the innovations admit identification of all structural SVAR matrices and impulse response functions, without the need for conventional *a priori* parameter restrictions. We give mathematical identification conditions and propose a flexible Gibbs sampling approach for the posterior inference on the MS-SVAR parameters.

The new methodology is validated using the US, EA and Estonian macroeconomic series, where the effects of different shocks are examined. For the US data, we estimate the short-run impacts and impulse responses of four structural shocks, which we label a "monetary policy shock", a "money demand shock", an "aggregate demand shock" and an "aggregate supply shock". We find a robust and well-pronounced "price puzzle" in response to a "monetary policy shock", while the "money demand shock" induces a strong reaction in US real output and prices. The application of our statistical shock identification methodology to the EA data delivers a full set of discernible structural shocks, which we label an "aggregate demand shock", an "aggregate supply shock" and a discretionary "monetary policy shock". As with the US data, the estimated posterior responses of the EA macroeconomic aggregates are found to be in line with the empirical literature. Finally, in application to the Estonian economy, we uncover three structural shocks, which appear to fit a theoretically sound narrative of exogenous "risk premium", "aggregate demand" and "aggregate supply" shocks. We find that an exogenous increase in the interest rate premium in the Estonian data leads to a strong and persistent negative effect on real output, while the inflation response remains muted.

We also point out that the results of our statistical shock identification procedure are not always compatible with traditional short run and sign identification schemes used in much of the recent literature on SVAR models, which warrants further careful validation and checking of the existing results using the new MS-SVAR methodology and other alternative approaches.

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