Eesti Pank Bank of Estonia



# Constants do not stay constant because variables are varying

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#### Rasmus Kattai\*

#### **Abstract**

This paper focuses on the dynamic properties of error correction models (ECM). It is shown that the absence of structural breaks in the cointegrating vector does not necessarily imply that also all parameters of the dynamic specification of the ECM are time invariant. In some cases, depending on the data generating process of regressors, the intercept has to be time varying in order to have the long run equilibrium of a dynamic model independent of the growth rates of the variables out of sample period, i.e. to satisfy the dynamic homogeneity condition. It is found to be common when estimating ECMs on macroeconomic time series of converging countries. Dynamic homogeneity can be achieved by imposing the state dependent dynamic homogeneity restriction on the intercept. Applying the restriction is illustrated by an empirical example using Estonian data on real wages and labour productivity.

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# **Non-technical summary**

An error correction model (ECM) serves as a useful tool for forecasting, because it distinguishes between the equilibrium value of macroeconomic time series and short run dynamics. The explicit representation of the equilibrium relationship produces the "backbone" of the modelled time series, which offers better understanding of the long run behaviour of the series. But as shown in the paper, if the equilibrium relationship combines time series, which have a non-constant expected mean growth rate, the equilibrium solution becomes dependent on the growth rates of the explanatory variables. The latter is the violation of the dynamic homogeneity condition (DHC). From the forecaster's perspective, the violation of DHC also means that the equilibrium equation and the dynamic equation project different growth rate of the modelled time series. This inconsistency may result in having substantial forecast errors.

As argued in the paper, a large share of macroeconomic time series of converging economies are generated by processes, which have a non-constant expected mean growth rate. As an example, one could think of an economic growth rate, which is, in accordance with economic theory, decreasing as the economy matures and reaches a steady state. Therefore ECMs, which are freely estimated on macroeconomic data of converging economies naturally violate DHC out of sample period. The paper develops a dynamic homogeneity restriction (DHR), which is set on the intercept of the dynamic equation in order to make DHC to hold and to avoid the related problems. The novelty of the approach is that the current method relates the DHR to the phase of economic development of a country, which makes the intercept time varying.

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#### 1. Introduction

An error correction model (ECM) is a standard specification of behavioural equations in macro-econometric models that are used for various purposes. In principle, ECM type models are very useful tools to produce long run forecasts as they contain an equilibrium relationship (cointegrating equation) in the error correction part. For the sake of forecast accuracy it is crucial that the parameters of the cointegrating relationship would hold over the forecast period. It is not surprising that there is a growing literature on structural breaks in cointegration vectors, extending the initial framework by Engle and Granger (1987). The related literature is motivated by the fact that long run constraints of cointegrated variables may vary in line with taste, technology, economic policy, institutional framework etc. By nature, equilibrium relationships between macroeconomic time series of converging economies are subject to this kind of structural shifts as these countries go through numerous reforms, restructuring, liberalization of markets etc.

Even if no breaks in the parameters of the cointegrating equation occur, another problem related to parameter stability may emerge. Namely, the nature of the catching up process implies a decreasing economic growth rate and a slowing inflation rate as the income level and the price level converge to those of more advanced countries. This is the pattern which is reflected in many other macroeconomic time series as well. In line with Johnston and DiNardo (1997), it is shown in the paper that estimating ECMs on this data may cause the violation of the dynamic homogeneity condition (DHC) out of sample period — the equilibrium solution becomes dependent on the growth rates of the explanatory variables. Equivalently, from the forecasting point of view the dynamic part of the ECM becomes inconsistent with the equilibrium specification, implying different growth rates of the endogenous variable, which increases the forecast error. The issue rises when producing long run forecasts and also when testing the long run properties of macro-econometric models and the convergence of key variables to their steady state ratios. The problem could be overcome by imposing a dynamic homogeneity restriction (DHR) on the intercept of the dynamic equation. Relating the restriction to the catching up process, the intercept becomes time varying.

The concept of DHC is briefly discussed in the next section, followed by deriving the restriction that makes dynamic homogeneity to hold at any time period. The fourth section provides an empirical example of using the restriction and the fifth section concludes.

# 2. Dynamic homogeneity condition

The idea of DHC states that the long run equilibrium solution of the ECM should be independent of the growth rates of the explanatory variables. Lets consider the following ECM:

$$A(L)\Delta y_t = c + \mathbf{B}^T(L)\Delta \mathbf{x}_t - \phi(y_{t-k} - \boldsymbol{\gamma}^T \mathbf{x}_{t-k}) + \epsilon_t, \tag{1}$$

where y is natural logarithm of variable Y (logarithms are denoted with the lower case letters),  $\mathbf{x}$  is the  $N \times 1$  vector of natural logarithms of explanatory variables  $X_n$ , n=1,...,N. A(L) is the lag polynomial such as  $A(L)=1-\alpha_1L-...-\alpha_gL^g$ , c is the intercept of the dynamic equation,  $\mathbf{B}(L)$  is  $N \times 1$  vector of lag polynomials, such as  $B_n(L)=\beta_{n,0}+\beta_{n,1}L+...+\beta_{n,h}L^h, -\phi(y_{t-k}-\boldsymbol{\gamma}^T\mathbf{x}_{t-k})$  is the error correction term and  $\epsilon_t \sim iid(0,\sigma^2)$  is the disturbance term.

The cointegrating vector  $[1, -\gamma^T]$  implies that the equilibrium growth rate of the endogenous variable is  $\gamma^T \Delta \mathbf{x}_t + \Delta \varepsilon_t$ , where  $\varepsilon_t \sim iid(0, \sigma^2)$  is the error term of the cointegrating equation. Denoting the vector of steady growth rates of  $X_n$  with  $\mathbf{g} = (g_1, ..., g_n)^T = \Delta \mathbf{x}_t$  for all t gives  $\gamma^T \mathbf{g}$  as the growth rate of Y. Substituting it in equation 1 yields the dynamic equilibrium of Y:

$$y = \frac{c - [A(L)\gamma - \mathbf{B}(L)]^T \mathbf{g}}{\phi} + \gamma^T \mathbf{x}.$$
 (2)

Equation 2 implies that the equilibrium level of Y is a function of growth rates of  $X_n$ , g. Botas and Marques (2002) argue that the steady state solution of the model should not respond to a shift in the "average" growth rates of the variables of the model, caused for example by changes in monetary policy or by an exogenous shock. Often the independence of the equilibrium solution is concerned in the context of inflation neutrality in the long run. For example, if the ECM for employment does not contain inflation neutrality (in equation 2, Y would be employment and at least one of the regressors  $X_n$  would be a nominal variable), unemployment could be changed by simply moving the inflation rate, which contradicts economic theory.

Alternatively expressed — DHC is guaranteed if the dynamic specification and the cointegration relationship of the ECM imply the same growth rate of the endogenous variable. In order to see under which conditions this requirement is satisfied,  $\Delta \mathbf{x_t}$  in equation 1 is replaced by the vector of steady growth rates  $\mathbf{g}$ ,  $\Delta y_t$  is substituted with  $\boldsymbol{\gamma}^T \mathbf{g}$  and taking expectations —  $E(\epsilon_t) = 0$  and

 $E(y_{t-k} - \gamma^T \mathbf{x}_{t-k}) = 0$ , gives that the cointegrating relationship is consistent with the dynamic specification only if

$$[A(L)\gamma - \mathbf{B}(L)]^T \mathbf{g} = c. \tag{3}$$

The above is the expression of DHC for the ECM as it was defined in equation 1. The same result could be obtained from equation 2, because it is known from the cointegrating relationship that  $y = \gamma^T \mathbf{x}$ . Based on equation 3, if any of  $g_n$  is not constant, then c has to be estimated as a time dependent parameter (if not an alternative approach is chosen, like reparameterising the model, as shown in section 3).

As stated above, changes in g could be caused, for example, by monetary policy and exogenous shocks. But these changes also could be systematic (endogenous) associated with the catching up process of the less developed countries — in line with economic theory macroeconomic time series of converging economies tend to have decreasing mean growth rate.<sup>1</sup>

# 3. State dependent dynamic homogeneity restriction

Growth models, starting with neoclassical paradigm by Cass (1965), Koopmans (1965), Solow (1956) and later the endogenous growth theory by Romer (1986) and Lucas (1988) state that poor countries tend to catch up with richer countries and also that economic growth is inversely correlated with the income level of a country. Consequently, a growth convergence is expected to take place. The related empirical literature finds support to this phenomenon (for example Barro (1991), Islam (1995) and Mankiw et al. (1992)).

Concentrating on the ten new EU member states (NMS) a clear connection between countries' income level and economic performance can be witnessed (see Figures 1 and 2). Data of the last decade shows convergence towards the EU-15 as also found by Feldkircher (2006). According to the regression line, an increase in relative income decreases the economic growth rate with almost unitary elasticity. The regression suggests that the annual growth rate

<sup>&</sup>lt;sup>1</sup>DHC becomes more complicated, if the dynamic specification of the ECM also includes variables, which are not present in the error correction term. Most common examples in the empirical modelling are, as stated by Botas and Marques (2002), the inclusion of unemployment rate and output gap to explain the short run dynamics of wages and inflation respectively (see Brouwer and Ericsson (1995) for empirical application).

of a country decreases to about two per cent after income has converged to the level of EU-15. Relying on a single country data, short time series may not allow to detect downward sloping growth rates. And that is also the case with the NMS (see Figure 3). But considering that the NMS are a homogeneous group of countries (given the EU membership that relates to the harmonisation of legal systems, free trade and factor movement, the perspective of joining the monetary union etc.), the cross-country evidence provides an insight of how each of these countries is developing. Based on the mean values across the selected countries, the output of a country can be described by the following data generating process (DGP):

$$x_t = \mu + \rho s_t + x_{t-1} + v_t, \quad \mu, \rho > 0 \quad v_t \sim iid(0, \sigma^2)$$
 (4)

where  $x_t$  denotes the log of output.  $s_t = [0,1]$  is a variable related to the state of economic development. In the current context  $s_t$  is a gap in the relative per capita income level —  $s_t = 0$  indicates that the gap is closed and the economy has reached a steady state. Writing equation 4 in differences allows us to see that the output growth is given by  $\Delta x_t = \mu + \rho s_t + v_t$  and it converges to  $E(\Delta x_t | s_t = 0) = \mu$ . It is important to note here that although according to equation 4 the expected mean value of  $\Delta x_t$  is not constant, it does not necessarily imply that  $x_t$  is I(2). In what follows  $x_t$  is defined as I(1) process.

Not only the output but also many other macroeconomic time series of converging economies are described by the same type of DGP. Besides direct links between economic growth and the related variables, one could think, for example, of the Balassa-Samuelson effect (Balassa, 1964), which provides theoretical concept of the connection between real and nominal variables. In the NMS price levels are positively correlated with income levels: a one per cent increase in relative income causes an increase in relative price level about 0.7 per cent (see Figure 4). If income and price levels are correlated, then the slowdown in economic growth would imply that also inflation behaves in the same manner. Having evidence on the negative correlation between the relative income level and the growth rate, prices are generated by process (4) as well — nominal convergence not only implies that price levels of countries are about to equalize but also inflation rates.

As a second step, the following bivariate cointegrated system of variables  $x_t$  and  $y_t$  is defined:

$$y_t = \gamma x_t + u_t, \quad u_t \sim iid(0, \sigma^2),$$
 (5)  
 $x_t = \mu + \rho s_t + x_{t-1} + v_t.$ 

Both  $y_t$  and  $x_t$  are I(1) processes and cointegrated with  $[1, -\gamma]$ . The presence of the cointegrated relationship can be verified by showing that the linear combination of  $y_t$  and  $x_t$  produces stationary time series  $h_t$ :  $h_t = \gamma x_t + \theta \mu + \theta \rho s_t + \theta x_{t-1} + u_t + \theta v_t$ , from which  $h_t = (\gamma + \theta)(x_{t-1} + \mu + \rho s_t) + u_t + (\gamma + \theta)v_t$ .  $h_t$  is stationary if  $\theta = -\gamma$ . Differentiating equation 5 and substituting  $x_t$  with equation 4 gives the following ECM presentation of the model:

$$\Delta y_t = c_t - (y_{t-1} - \gamma x_{t-1}) + u_t + \gamma v_t, \tag{6}$$

where  $c_t = \gamma(\mu + \rho s_t)$ . The latter is what is called here a state dependent dynamic homogeneity restriction as the intercept is a function of the phase of economic development, captured by  $s_t$ . Notice that if  $s_t$  stays time invariant (if the economy is considered to be on a balanced growth path or fairly close to it),  $c_t$  can be estimated as a constant. But ignoring the possible shift in  $c_t$  may lead to considerable forecast errors, as shown later. Interestingly deriving the DHR sets the restriction only on the intercept of the dynamic equation. In economic reasoning, the intercept of the dynamic equation is interpreted as an "autonomous" growth rate or a growth rate that is not captured by the regressors. Therefore one could expect that all dynamic parameters would adjust equally to make the DHC to hold.

Drawing a parallel with time varying parameters of the cointegrating vector, Clements and Hendry (1996) state that if models error-correct on the outdated cointegration structure, it leads to a biased forecast. They argue that if there is only a one-off change in the intercept of the DGP occurring in the sample period, then the optimal solution in the sense of yielding unbiased forecasts would be adding a residual of the period each step ahead. Alternatively, Hendry and Mizon (2001) suggest "intercept corrections" if the DGP is not known. But shifts in the intercept of the dynamic specification can be viewed as of the systematic pattern instead of one-time type of regime shifts or structural breaks. Being related to the DGP of regressors, these shifts can be taken into account by imposing a DHR, which would eliminate the forecast bias originating from the misspecification of the dynamic equation.

In order to prove that equation 6 satisfies the dynamic homogeneity condition  $\Delta y_t$  on left hand side is replaced with  $\gamma \Delta x_t + \Delta u_t$ , which is the long run growth rate implied by the cointegrating equation 5. Thereafter substituting  $\Delta x_t$  with  $\mu + \rho s_t + v_t$  and taking expectations of both sides yields, after cancelling,  $E(v_t) = 0$ , which holds because  $v_t \sim iid(0, \sigma^2)$ .

The restriction in a more general form, taking the specification of the ECM as given in equation 1 and defining  $\mathbf{x}_t$  as  $\boldsymbol{\mu} + s_t \boldsymbol{\rho} + \boldsymbol{v}_t$  ( $\boldsymbol{\mu} = (\mu_1, ..., \mu_N)^T$ ) and  $\boldsymbol{\rho} = (\rho_1, ..., \rho_N)^T$ ), becomes:

$$c_t = [A(L)\gamma - \mathbf{B}(L)]^T E(\boldsymbol{\mu} + s_t \boldsymbol{\rho}). \tag{7}$$

Given any specification of the ECM, it is relevant to uncover the DGP of  $\mathbf{x}_t$  in order to set the restriction. As illustrated above by using NMS data, time series may not reflect the information on DGP because of the short sample period. Then it is optional to use a cross-country analysis to calibrate the parameters of the DGP (here  $\mu$  and  $\rho$ ). Alternatively the DHR can be written down as:

$$c_t = [A(L)\gamma - \mathbf{B}(L)]^T E \Delta \overline{\mathbf{x}}_t, \tag{8}$$

having  $\Delta \overline{\mathbf{x}}_t$  the vector of moving average growth rates of regressors. Using moving average values is suggested by the time series being often too volatile. But even more importantly, the expression  $\boldsymbol{\mu} + s_t \boldsymbol{\rho}$  captures long run growth trends, which is the idea of the whole concept. Using  $\Delta \overline{\mathbf{x}}_t$  instead of  $\Delta \mathbf{x}_t$  brings us closer to long run growth paths, leaving aside short run fluctuations in data. The merit of defining the DHR as in equation 8 is that there is no necessity to specify the DGP.

Botas and Marques (2002) also show that DHC could be satisfied by imposing a simpler restriction on dynamic parameters after reparameterising the model. They utilise the property of polynomials, which allows to define  $D(L) = \sum_{j=0}^{m} d_j L^j = D^*(L)(1-L) + D(1)L$ , where  $D(1) = \sum_{j=0}^{m} d_j$ ,  $d_0^* = d_0$ ,  $d_j^* = -\sum_{i=j+1}^{m} d_i$ , j = 1, ..., m-1. The property above enables to reparameterise the ECM in second order differences and equation 1 would take the form:

$$A^*(L)\Delta y_t^2 = c + \mathbf{B}^{*T}(L)\Delta \mathbf{x}_t^2 - A(1)\Delta y_{t-1} + \mathbf{B}^T(1)\Delta \mathbf{x}_{t-1}$$
 (9)  
 
$$- \phi(y_{t-k} - \gamma' \mathbf{x}_{t-k}) + \epsilon_t.$$

If  $\gamma$  is a vector of ones then imposing the restriction  $A(1) = B_n(1)$  for n = 1, ..., N in the reparameterised model makes the static and dynamic equilibrium solutions to coincide and DHC is satisfied. The restriction by Botas and Marques (2002) differs from the ones presented in (7) and (8) in two aspects — the restriction is not set on the intercept and all coefficients are time invariant. The relative advantage of the DHR by Botas and Marques (2002) is it that no explicit representation of the expected growth rates of the variables  $E\Delta \mathbf{x}_t$  is required. On the other hand, the possible shortcoming of the method is that the interpretation of the ECM becomes complicated, especially in the context of a macro-econometric model.

# 4. Empirical application

The state dependent dynamic homogeneity restriction is illustrated by estimating an ECM of real wage using Estonian data. The data period covers 1996 -2005 at quarterly frequency (36 observations). The time series are seasonally adjusted by TRAMO-SEATS<sup>2</sup>.

The equilibrium relationship is derived based on the neoclassical growth theory in which labour earns its marginal product revenue. Differentiating representative firm's profit maximization function with respect to labour yields:

$$w_t^* = \ln[(1-a)/\eta] + z_t, \tag{10}$$

where  $w_t^*$  is the equilibrium level of real wage (as the error term is omitted), 1-a is the labour share of income,  $\eta$  denotes the mark up and  $z_t$  is GDP per labour (all variables are presented in logarithms). Parameter values of a=0.37 and  $\eta=1.12$  are taken from the Bank of Estonia's macro model of the Estonian economy (Kattai, 2005). The analysis of the error term shows that it is stationary (see Figure 5). Therefore equation 10 is appropriate to describe the equilibrium level of real wage, according to which real wage grows at the rate of labour productivity in the long run. Possible structural breaks are set aside in the equilibrium relationship and do not test for them.

The ECM for real wage is constructed as shown in equation 1 and the lag structure is specified depending on parameter significance. The estimated equation takes the form:

$$\Delta w_t = \varphi_0 + \varphi_1(w_{t-3} - w_{t-3}^*) + \varphi_2 \Delta z_{t-2} + \varphi_3 \Delta z_{t-3}$$

$$+ \varphi_4 \Delta w_{t-1} + \varphi_5 \Delta w_{t-3} + \zeta_t.$$
(11)

In what is presented above  $\zeta_t$  is the error term. Three equations are estimated in parallel by the OLS — one without and two others with the DHR. In the first case  $\varphi_0$  is estimated by treating it as a constant. In the second case a state dependent DHR, as given in equation 7, is applied. In the third case a DHR in the form of equation 8 is used. To derive both of the restrictions it is considered that according to the cointegrating equation the expected real wage growth equals labour productivity growth:  $E\Delta w_t = E\Delta z_t, \forall t$ .  $E\Delta w_{t-n} \equiv L^n E\Delta w_t \equiv E\Delta w_t$  is assumed to hold if n is a small number.

<sup>&</sup>lt;sup>2</sup>Time Series Regression with ARIMA Noise, Missing Observations and Outliers, SEATS
— Signal Extraction in ARIMA Time Series.

Analogously  $E\Delta z_{t-n} \equiv L^n E\Delta z_t \equiv E\Delta z_t$ , where L is the lag operator. The restrictions become:

$$\varphi_{0,t} = (1 - \varphi_2 - \varphi_3 - \varphi_4 - \varphi_5)(\mu + \rho s_t),$$
 (12)

$$\varphi_{0,t} = (1 - \varphi_2 - \varphi_3 - \varphi_4 - \varphi_5) \Delta \overline{z}_t. \tag{13}$$

From the above it is known that  $\Delta z_t$  is generated by the process  $\Delta z_t = \mu + \rho s_t + v_t$ ,  $v_t$  being the disturbance. The cross-country analysis of NMS suggests that  $\mu = 0.005$  and  $\rho = 0.019$ , which means that the quarterly growth rate diminishes by about 0.02 percentage points if the gap between Estonian and EU-15 per capita income levels closes by one percentage point. The quarterly growth rate of output per labour converges to 0.005 if  $s_t$  converges to zero (the sample mean value is 0.016). To assess the validity of the DGP, it is checked whether  $\Delta \overline{z} = \mu + \rho \overline{s}$  holds in the sample period, where  $\overline{z}$  and  $\overline{s}$  are the sample mean values of  $z_t$  and  $s_t$  respectively. The data confirms the validity of the DGP — the mismatch in the actual mean growth rate and the one given by the DGP is only 0.05 percentage points (0.0005).

Comparing three specifications, estimation results show no significant changes in dynamic coefficients (see Table 1). As expected the fit of regressions with DHR is lower. The fall in the fit is larger in the case of specification (C), which is due to the higher variance of  $\varphi_{0,t}$  compared to (B) (see Figure 6). The residuals of equations show similar pattern (see Figure 7).

Table 1: Estimation results

	Without DHR	With DHR	With DHR
	(A)	(B)	(C)
$\hat{\varphi}_0$	0.009***	$\varphi_{0,t} = f(\mu, \rho, s_t)$	$\varphi_{0,t} = f(\Delta \overline{z}_t)$
$\hat{arphi}_1$	$-0.169^{***}$	$-0.210^{***}$	$-0.217^{***}$
$\hat{\varphi}_2$	0.158***	0.183***	0.183***
$\hat{\varphi}_3$	$0.197^{**}$	$0.177^{**}$	$0.196^{**}$
$\hat{arphi}_4$	$-0.607^{***}$	-0.579***	$-0.457^{**}$
$\hat{\varphi}_5$	$0.474^{***}$	$0.516^{***}$	$0.602^{***}$
$R^2$	0.666	0.632	0.484
$\sigma$	0.006	0.006	0.008
DW	1.983	1.891	1.974

Notes: \*\* and \*\*\* mark the statistical significance at 5% and 1% significance level respectively.  $\sigma$  denotes s.e. of regression. DW denotes Durbin-Watson statistic. Number of observations included: 32.

The appropriateness of the equations for long run forecasts is analysed by assessing the consistency of the equilibrium relationship and the dynamic specification.  $\tau$  is used to mark time period when Estonia's per capita income level reaches that of EU-15. Based on the earlier —  $E(\Delta z_t | t \geq \tau) = 0.005$ . The dynamic equation (A) gives for the real wage growth  $E(\Delta w_t | t \geq \tau) = [\hat{\varphi}_0 + (\hat{\varphi}_2 + \hat{\varphi}_3)E(\Delta z_t | t \geq \tau)]/(1 - \hat{\varphi}_4 - \hat{\varphi}_5) = 0.009$ , which is in contradiction with the growth rate implied by the cointegrating equation because it states  $E\Delta w_t = E\Delta z_t, \forall t$ . The dynamic equation projects almost two times higher growth rate than the equilibrium relationship. As a consequence, the ECM fails and  $\Delta w_t$  as well as  $w_t$  may permanently diverge from its equilibrium level.

Proceeding analogously with equation (B), the expected real wage growth becomes  $E(\Delta w_t|t\geq\tau)=[\hat{\varphi}_{0,t}+(\hat{\varphi}_2+\hat{\varphi}_3)E(\mu+\rho s_t|t\geq\tau)]/(1-\hat{\varphi}_4-\hat{\varphi}_5).$  Substituting  $\hat{\varphi}_{0,t}$  with the expression 12 and having  $E(s_t|t\geq\tau)=0$  gives  $E(\Delta w_t|t\geq\tau)=\mu.$  A similar procedure gives for equation (C)  $E(\Delta w_t|t\geq\tau)=E(\Delta z_t|t\geq\tau).$  Both of these hold because  $E(\Delta w_t|t\geq\tau)=E(\Delta z_t|t\geq\tau)=E(\Delta z_t|t\geq\tau)=\mu.$  The above shows that the ECM type of equations in the macro models of converging economies and taken separately must be restricted by the DHR. Otherwise these models are not appropriate for long run simulations and produce faulty characterisations of the development of an economy.

#### 5. Conclusions

In this study the issue of parameter stability in the dynamic specification of ECMs was addressed. The finding of the paper was that if the data generating process of regressors projects change in the expected growth rate, as it is common in case of many macroeconomic time series of converging economies, an intercept of the ECM dynamic equation has to be modelled as a time varying parameter in order to guarantee that the DHC would hold. The issue becomes relevant when the ECM under discussion is used for long term forecasting. If the DHC is violated, the forecast of the modelled variable will permanently diverge from the equilibrium given by the long run relationship. A state dependent DHR is provided to guarantee that the DHC would be satisfied at any time period.

The field of research offers several possibilities for future developments. The time variance of the intercept is mostly related to the macroeconomic time series of catching up economies. Most likely these economies also face structural shifts in the equilibrium relationship. Therefore it would be interesting to investigate how dynamic parameters should be restricted in the presence of breaks of the cointegrating equation in order to fulfill the DHC.

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# **Figures**

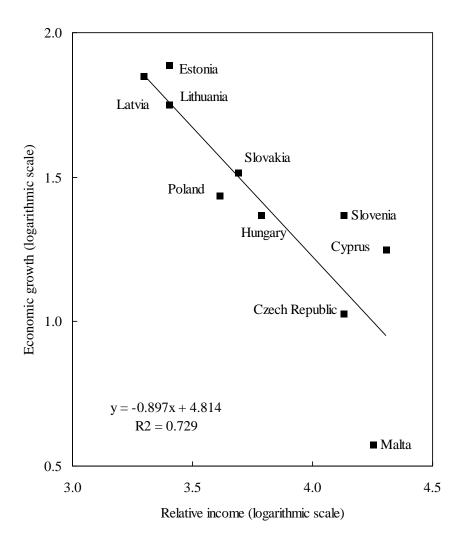


Figure 1: Income level and economic growth in NMS (relative income: GDP per capita in PPS EU-15 = 100 (1995, except Malta — 1998); economic growth: yearly GDP growth (1995–2006 average)).

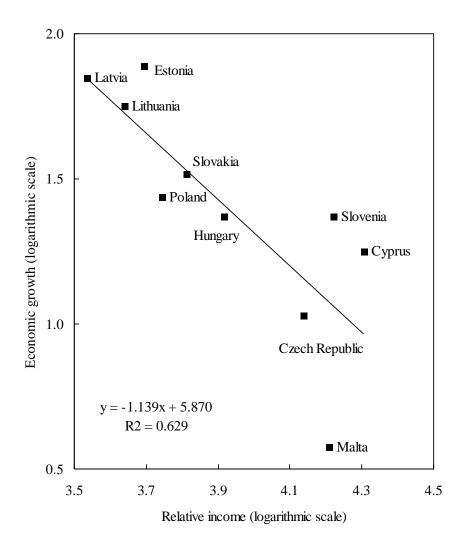


Figure 2: Income level and economic growth in NMS (relative income: GDP per capita in PPS EU-15 =100 (1995–2006 average, except Malta — 1998–2006 average); economic growth: yearly GDP growth (1995–2006 average, except Malta — 1998–2006 average).

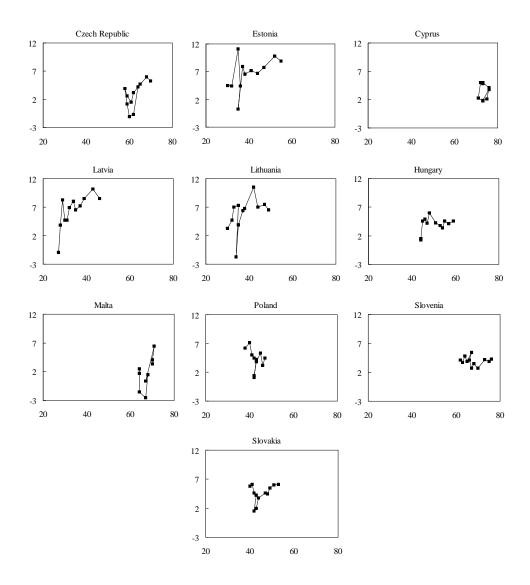


Figure 3: Income level and economic growth in NMS 1995–2006 (vertical axis: yearly GDP growth; horizontal axis: GDP per capita in PPS EU-15 = 100).

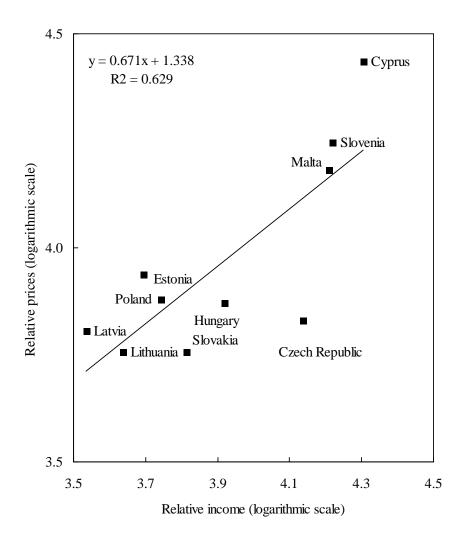


Figure 4: Income and price levels in NMS (relative income: GDP per capita in PPS EU-15 = 100 (1995–2006 average); relative prices: comparative price level EU-15 = 100 (1997–2004 average)).

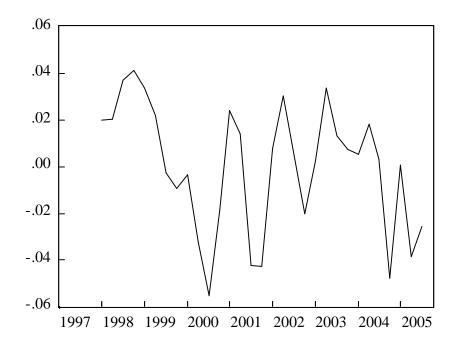


Figure 5: Residuals of the equilibrium relationship of real wage.

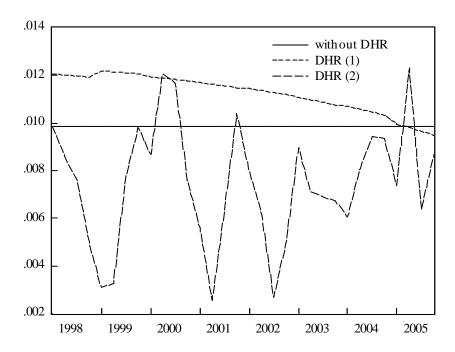


Figure 6: The estimated intercept and dynamic homogeneity restrictions. DHR (1):  $\varphi_{0,t}=f(\mu,\rho,s_t)$ , DHR (2):  $\varphi_{0,t}=f(z_t)$ .

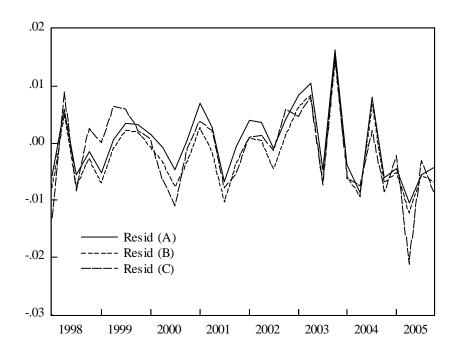


Figure 7: Residuals of the error correction models (A), (B) and (C).

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